Rotational Magneto-Acousto-Electric Tomography: Theory and Experiments

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Traditional modalities (please, don't take this slide literally!)

Type	Costs	Purpose	Mathematics	Instability
X-Ray	expensive	bone structure	Radon transform	mild
Gamma	expensive	blood flow	Attenuated Radon	stronger
Acoustic	cheap	soft tissue	Born/Rytov	mild
Microwave	cheap	high contrast	non-linear ?	???
MRI	very expensive		Fourier transform	mild
Impedance	very cheap	lung motion ?	divergence eq-n	very strong
Optical	cheap	small objects	diffusion eq-n ?	strong ?

Hybrid methods: motivation

Conductivity in tumors is much higher than that in healthy tissues
$\Longrightarrow$ EM waves or currents yield high contract.

Electrical impedance tomography, optical and microwave tomographies lead to strongly non-linear and Il-posed inverse problems = **BAD!**

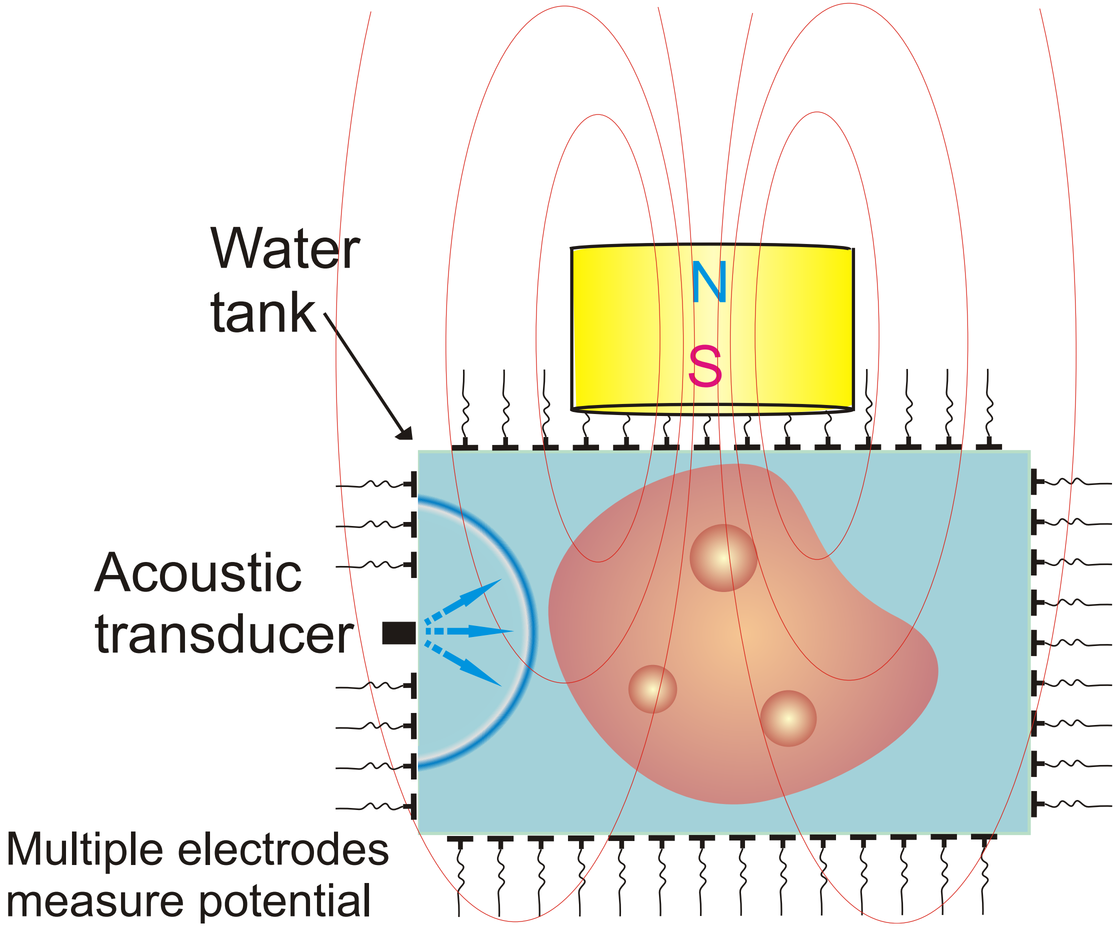
Acoustic waves yield high resolution but the contrast is low.

Idea: Use hybrid techniques, couple ultrasound with EM field:

Thermo-Acoustic and Photo-Acoustic Tomography (TAT/PAT)
Ultrasound Modulated Optical Tomography (UMOT)
Acousto-Electric Tomography (AET)
Magneto-Acousto-Electric Tomography (MAET)
Magneto-Acoustic Tomography with Magnetic Induction (MAT-MI)

Some of these techniques are "theoretical"

Lorentz Force Tomography (a.k.a MAET)



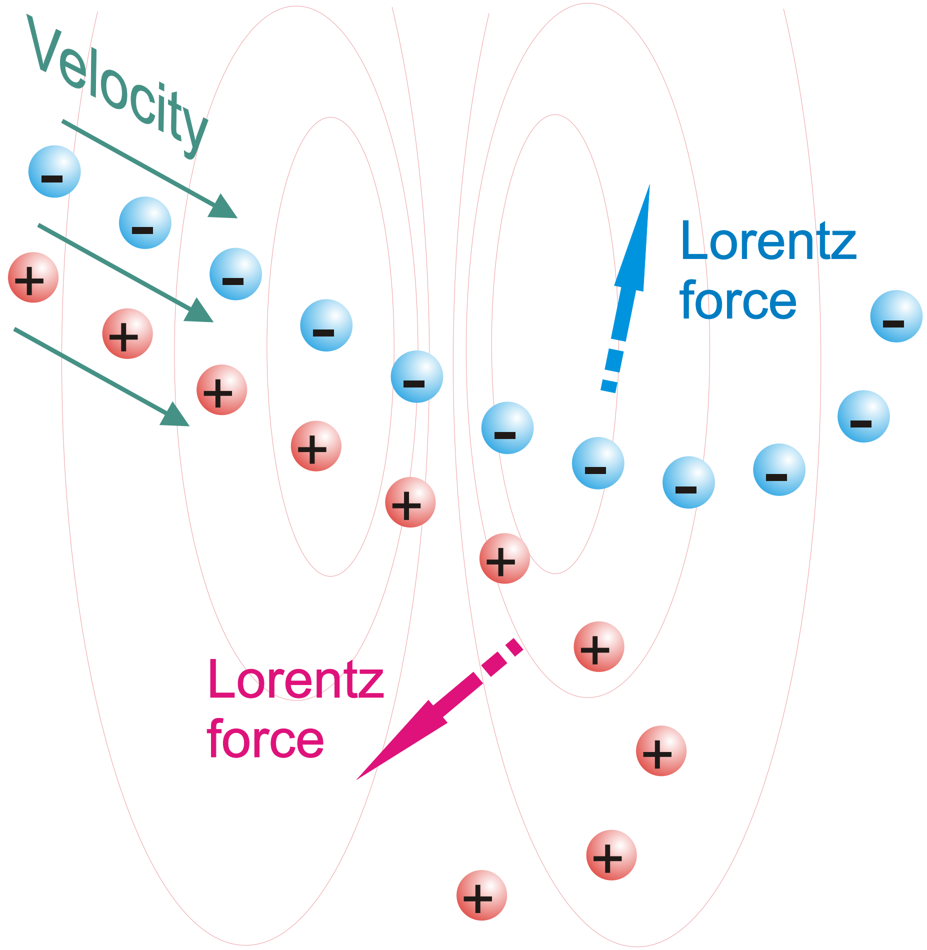
Ultrasound makes electrones and ions vibrate.

As a result, moving electrons and ions are separated by the Lorentz force.

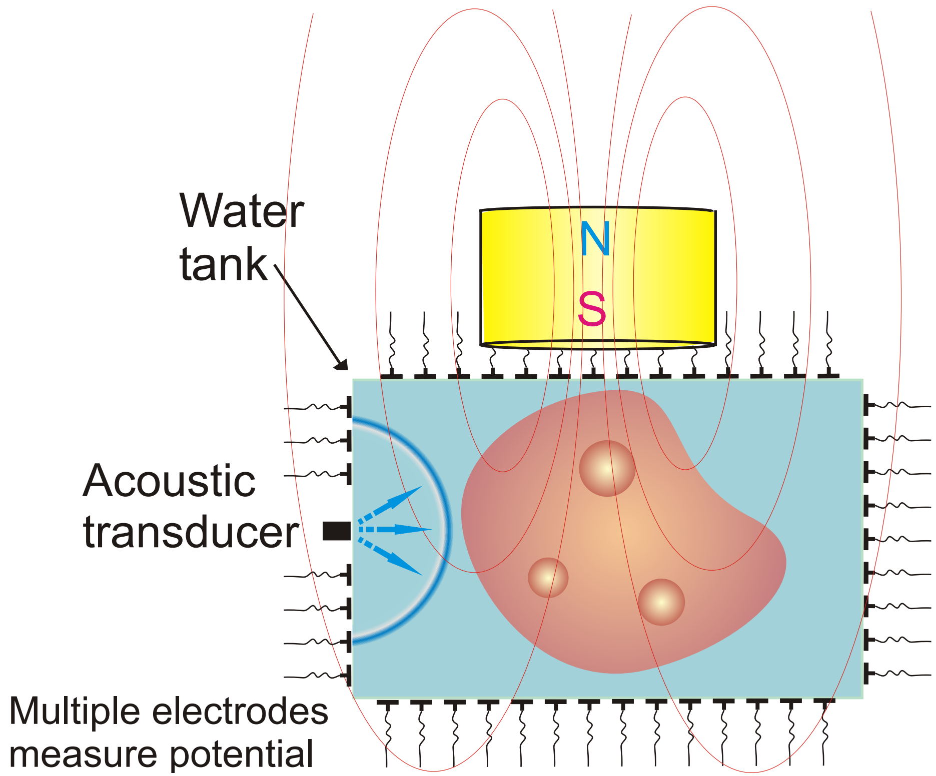
What's the Lorentz Force?

In a magnetic field the Lorentz force pushes moving charges sideways

Positive and negative particles are pushed in the opposite directions



MAET (a.k.a Lorentz Force Tomography)



Separated charges create an electric field that’s picked up by the electrodes

With some clever mathematics one can reconstruct an image

Previous work on MAET

The mathematics of MAET (partially explaind below) is very promising

MAET signal has been demonstrated only in one-directional measurements

No truly tomographic MAET images have been obtained before

Our goal: to demonstrate the feasibility of a full-scale MAET

Physics & mathematics of MAET

Tissue moving with velocity $V(x,t)$ produces Lorentz currents $J_L(x,t)$:

$$J_L(x,t) = \sigma(x) B \times V(x,t)$$

There will also be Ohmic currents satisfying Ohm's law

$$J_O(x,t) = \sigma(x) \nabla u(x,t).$$

There are no sinks or sources, the total current is divergence-free

$$\nabla \cdot (J_L + J_O) = 0.$$

Thus

$$\nabla \cdot \sigma \nabla u = -\nabla \cdot (\sigma B \times V).$$

BC: the normal component of the total current $J_L(x,t) + J_O(x,t)$ vanishes:

$$\left. \frac{\partial}{\partial n} u(z) \right|_{\partial\Omega} = -(B \times V(z)) \cdot n(z)$$

Measuring functionals

At any given time t we measure potential $u(z,t)$ at all $z \in \partial\Omega$.

Integrate boundary values of u with weight $I(z)$ and get a functional $M(t)$:

$$M(t) = \int_{\partial\Omega} I(z)u(z,t)dA(z),$$

Introduce lead currents = virtual currents

Consider lead potential $w_I(x)$ and lead current $J_I(x) = \sigma(x)\nabla w_I(x)$:

$$\nabla \cdot \sigma \nabla w_I(x) = 0,$$

$$\left.\frac{\partial}{\partial n} w_I(z)\right|_{\partial\Omega} = I\left(z\right).$$

Then, using the second Green's identity (= **reciprocity** principle):

$$M(t) = \int_{\Omega} B \cdot J_I(x) \times V(x,t)dx$$

Analyzing the velocity field

Assume that speed of sound c and density ρ are constant.

Then, velocity is the gradient of the velocity potential $\varphi(x,t)$:

$$V(x,t) = \frac{1}{\rho}\nabla\varphi(x,t),$$

where velocity potential $\varphi(x,t)$ is the time anti-derivative of pressure $p(x,t)$:

$$p(x,t) = \frac{\partial}{\partial t}\varphi(x,t).$$

Substitute into equation for $M(t)$ and integrate by parts:

$$M(t) = \frac{1}{\rho}B \cdot \left[\int_{\partial\Omega} \varphi(z,t)J_I(z) \times n(z)dA(z) + \int_{\Omega} \varphi(x,t)\nabla \times J_I(x)dx\right]$$

Volumetric part shows that we measure components of **curl** $J_I(x)$!

curl $J_I(x) = \nabla \times [\sigma(x)\nabla w_I(x)] = \nabla\sigma(x) \times \nabla w_I(x) = \nabla \ln \sigma(x) \times J_I(x)$

Notice: in the regions where $\sigma(x)$ is constant, **curl** $J_I(x) = 0$. No signal!

Reconstruction procedure

If $\varphi(x,t)$ could be focused into a point, i.e. $\varphi(x,0) = \delta(x - x_0)$, then

$$M_{x_0}(0) = \frac{1}{\rho} B \cdot \left[\int_\Omega \delta(x - x_0) \mathrm{curl} J_I(x) dx \right] = \frac{1}{\rho} B \cdot \mathrm{curl} J_I(x_0).$$

If three differenent directions of B are used, we have $C(x_0)$ = curl$J_I(x_0)$!

Chain of equtions to solve:

Curl C -> Current I -> $\nabla \ln \sigma(x)$ -> Conductivity $\sigma(x)$.

The second step comes from:

$$\nabla \ln \sigma \times J = C.$$

If we have two currents $J^{(j)}(x)$, $j = 1, 2$, then solve for $\nabla \ln \sigma$ at each x

$$\begin{cases} \nabla \ln \sigma(x) \times J^{(1)}(x) = C^{(1)}(x) \\ \nabla \ln \sigma(x) \times J^{(2)}(x) = C^{(2)}(x) \end{cases}.$$

3D MAET with ideal measurements

To summarize:

The inverse problems for MAET with ideal measurements is

stable

(almost) explicitly solvable...

... by a linear algorithm

(see [Kunyansky, 2012])

This is very rare, and very promising from the engineering standpoint.

No experimental work for 3D MAET has been ever done.

Something simpler: 2D MAET

Full 3-D scanner for MAET is difficult to build

We want to demonstrate the feasibility of MAET in a 2D setting

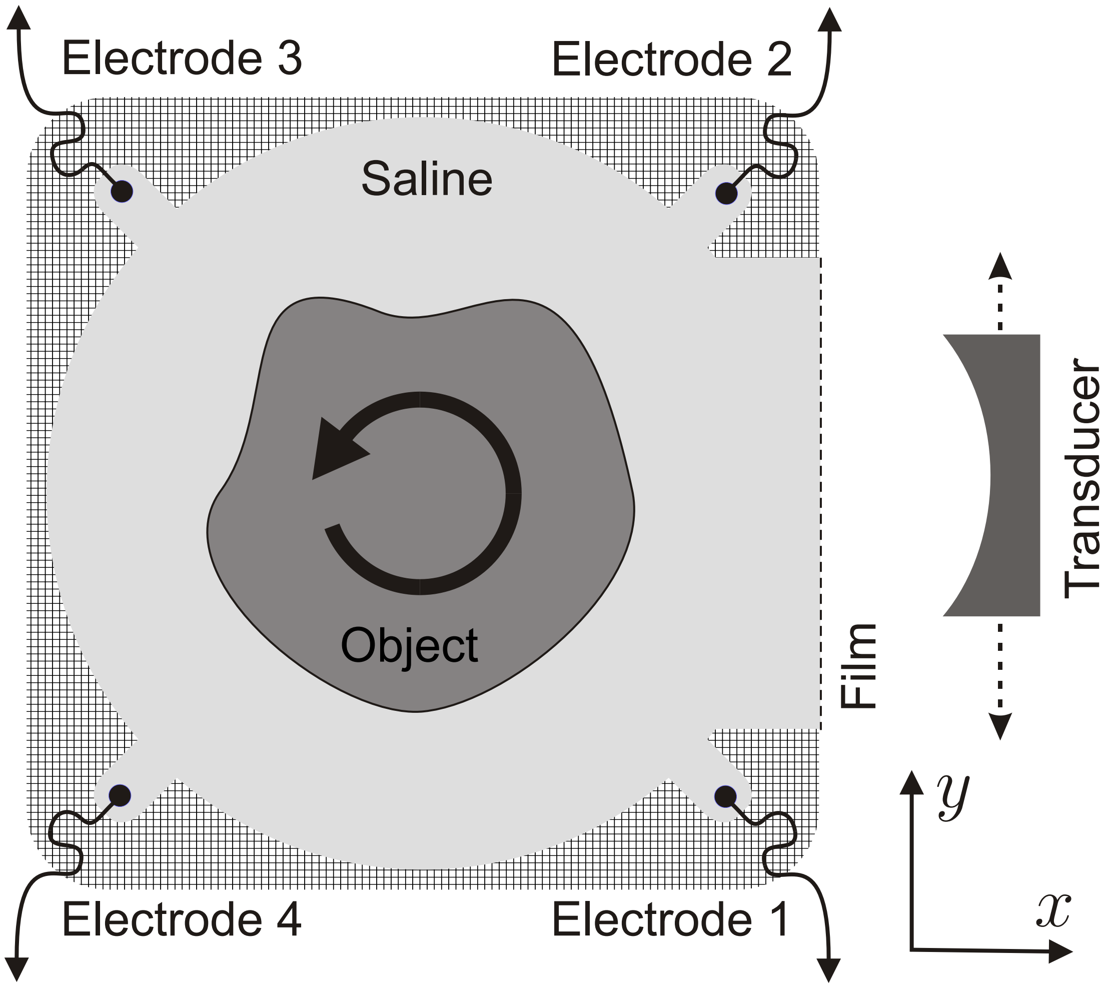
Assumptions and approximations:

- Everything is constant in the vertical direction ($\vec{e}_z$).
- Magnetic induction $B = b\vec{e}_z$ (vertical and constant).
- All the objects have vertical boundaries (genereralized cylinders)
- Electrodes are vertical lines

Then, all curls are vertical and parallel to B and we measure $\frac{b}{\rho}\text{curl}_z J$

2D MAET scanner

A simple 2D MAET scanner, top view:



Synthetic flat transducer

Problem:
The exact time-dependent velocity field of a focusing transducer is complicated and difficult to measure.

Instead, we average all measurements corresponding to a fixed angular position of the object and varying vertical position of the transducer.

Due to linearity of the problem, this is equivalent to using a large, flat, vertically oriented sound-emmiting source.

Approximately:

$$\varphi(t, x) = c_{\mathrm{tr}} \delta(-x_{\mathrm{tr}} + x_1 + ct),$$

and (if $C(x)$ is the curl)

$$M(t) = \frac{bc_{\mathrm{tr}}}{\rho} \int\limits_{\Omega} \delta(-x_{\mathrm{tr}} + x_1 + ct) C(x) dx = \frac{bc_{\mathrm{tr}}}{\rho} \int\limits_{\mathbb{R}} C\left[(x_{\mathrm{tr}} - ct)\vec{e}_1 + s\vec{e}_2\right] ds,$$

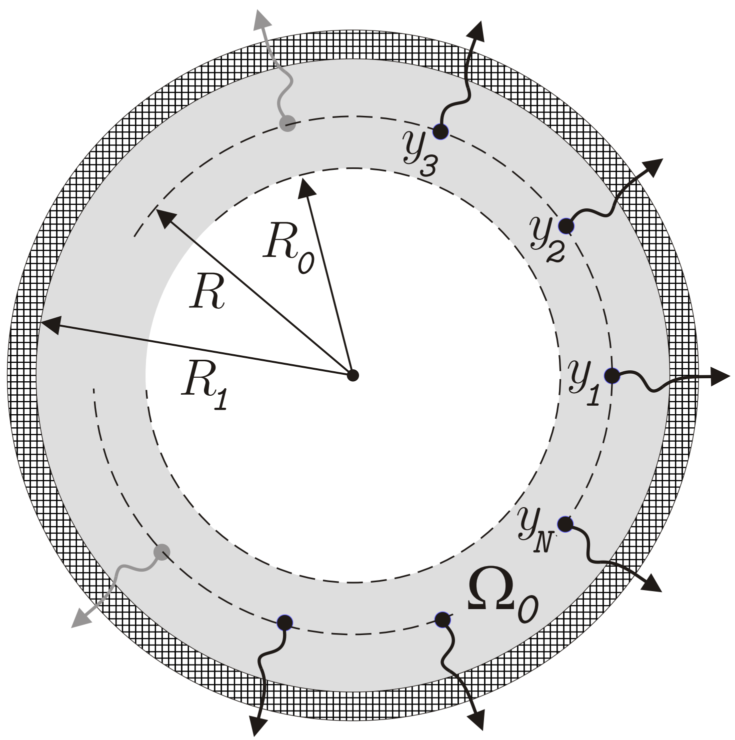
Thus, we measure **Radon projections** of $C(x)$ (integrals over vertical lines).

MAET with a rotating object

Problem:
We want to reconstruct curl $C(x)$ from its Radon projections.

However, if the object rotates, but the electrodes are stationary, the currents change and the curl $C(x)$ changes.

To resolve this, consider a round chamber, with N electrodes equispaced on a circle of radius R:



Lead currents and potentials; round chamber

The lead potential corresponding to a set of weights $\mathbf{W}$ equals:

$$w_{\mathbf{W}}(x) = w_{\mathbf{W}}^{\text{sing}}(x) + w_{\mathbf{W}}^{\text{smooth}}(x),$$

where $w_{\mathbf{W}}^{\text{sing}}$ is defined as

$$w_{\mathbf{W}}^{\text{sing}}(x) \equiv h(x) + \frac{1}{2\pi\sigma_0}\sum_{j=1}^{N} W_j \ln|x - y_j|,$$

with $h(x)$ harmonic in Ω and such that

$$\frac{\partial}{\partial n} w_{\mathbf{W}}^{\text{sing}}(z) = 0, \qquad z \in \partial\Omega.$$

Then $w_{\mathbf{W}}^{\text{smooth}}(x)$ is the solution of the following BV problem:

$$\nabla \cdot \sigma \nabla w_{\mathbf{W}}^{\text{smooth}}(x) = -\chi(x)\nabla \cdot \sigma(x)\nabla w_{\mathbf{W}}^{\text{sing}}(x), \qquad x \in \Omega.$$

$$\frac{\partial}{\partial n} w_{\mathbf{W}}^{\text{smooth}}(z) = 0, \qquad \in \partial\Omega,$$

Is $w_{\mathbf{W}}^{\text{smooth}}$ a "scattering of incoming potential $w_{\mathbf{W}}^{\text{sing}}$ by $\sigma(x)$" ?

Synthetic lead currents

For an arbitrary unit vector $\gamma = (\cos\alpha, \sin\alpha)$,
define the set of weights $\mathbf{W}^{\gamma} = (W_1^{\gamma}, ..., W_N^{\gamma})$ by the formula:

$$W_j^{\gamma} \equiv \frac{1}{N}\cos\left(\frac{2\pi(j-1)}{N} - \alpha\right).$$

Then

$$w_{\mathbf{W}^{\gamma}}^{\text{sing}}(x) \approx \beta\gamma \cdot x,$$

where β is a known constant.

This approximation converges exponentially in the limit $N \to \infty$.

Now, the corresponding lead potential $w_{\mathbf{W}^{\gamma}}$ can be synthetically rotated, by simultaneously turning the object and adjusting angle α.

The rest of the problem is solved explicitly, as before

The case of a realistic piezoelectric transducer

A big problem:

Widely used piezoelectric transducers do not reproduce low frequencies.

As a result, only a high-frequency component of $C(x)$ can be reconstructed.

Lead currents cannot be reconstructed at all.

A **very crude** solution: use near-constant approximation of $\sigma(x)$. Then:

$$w_{\mathbf{W}^\gamma}(x) \approx w^{\text{sing}}_{\mathbf{W}^\gamma}(x) \approx \beta\gamma \cdot x,$$

$$J(x) \approx \sigma_0 \beta\gamma,$$

$$C(x) = \sigma_0 \beta\gamma^\perp \cdot \nabla \ln \sigma(x)$$

Finally...

We use two orthogonal lead currents $J^{(1)} \approx \sigma_0 \beta \gamma^{(1)}$ and $J^{(1)} \approx \sigma_0 \beta \gamma^{(2)}$, with curls $C^{(1)}(x)$ and $C^{(2)}(x)$.

Then

$$C^{(1)}(x) \approx -\sigma_0 \beta \gamma^{(1)} \cdot \nabla \ln \sigma(x) = -\sigma_0 \beta \frac{\partial \ln \sigma(x)}{\partial \gamma^{(1)}},$$

$$C^{(2)}(x) \approx \sigma_0 \beta \gamma^{(2)} \cdot \nabla \ln \sigma(x) = \sigma_0 \beta \frac{\partial \ln \sigma(x)}{\partial \gamma^{(2)}},$$

and

$$\Delta \ln \sigma(x) \approx \frac{1}{\sigma_0 \beta} \left(\frac{\partial}{\partial \gamma^{(2)}} C^{(2)}(x) - \frac{\partial}{\partial \gamma^{(1)}} C^{(1)}(x) \right).$$

While this simple technique is based on a small perturbations of constant conductivity, in practice, it captures boundaries of material interfaces even if $\sigma(x)$ is strongly nonuniform

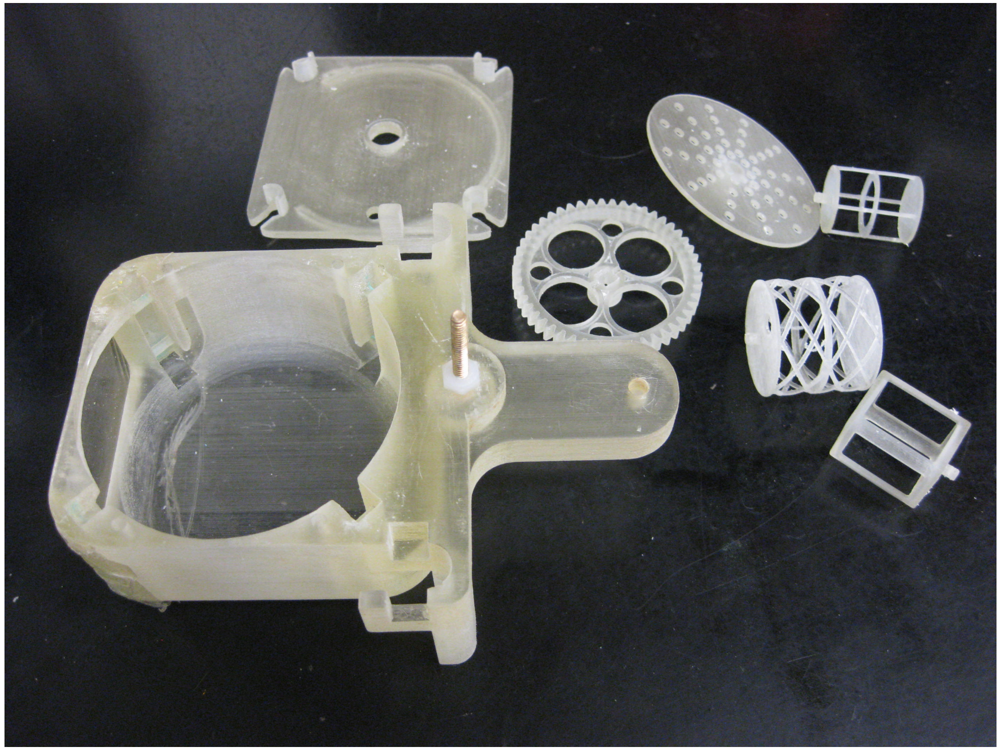
The experiment

Joint work with R. Witte and P. Ingram, Medical Imaging Department, UA

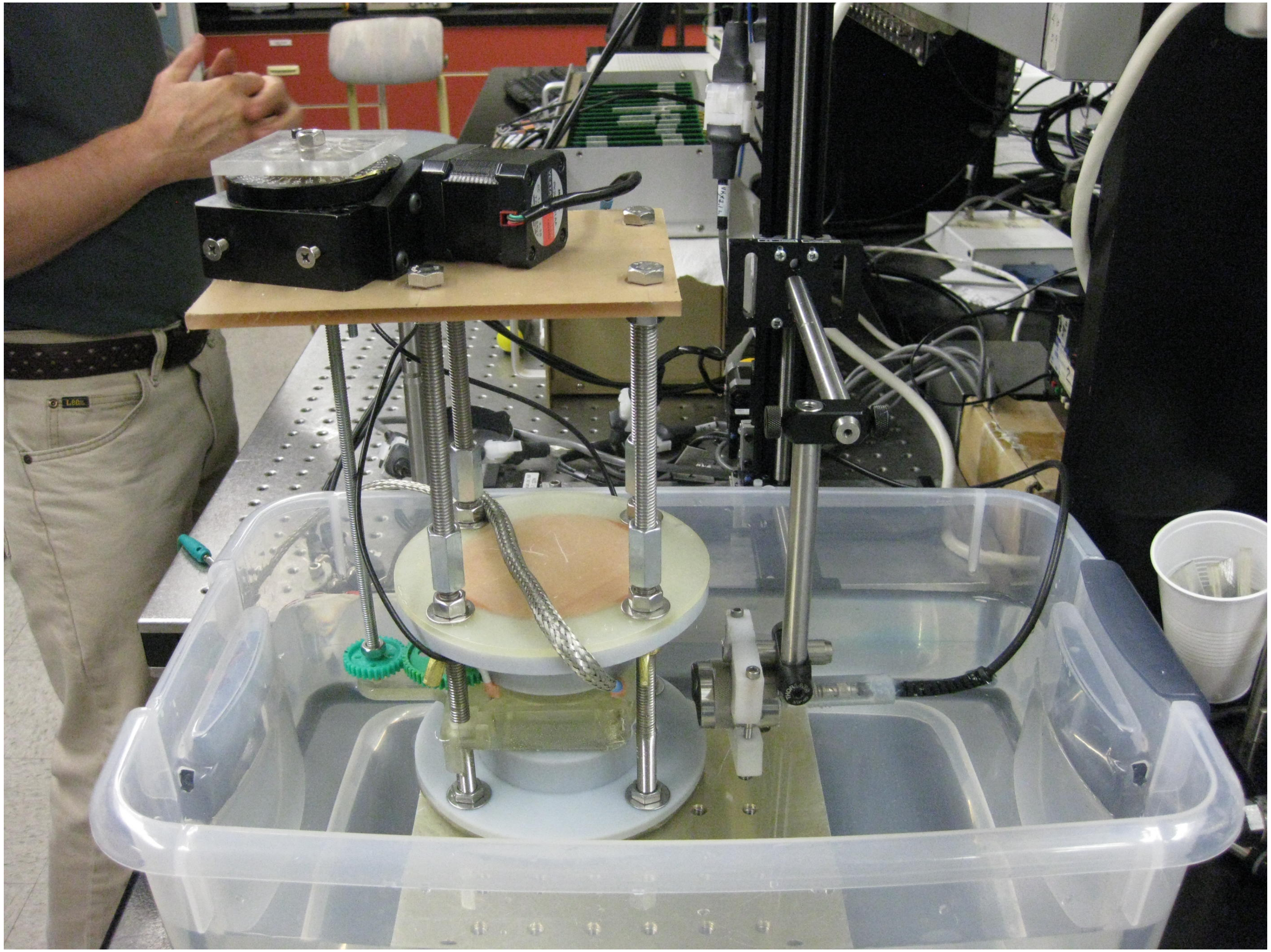
Supported by a BIO5 fellowship, but no money for hardware :(

Goal: build the first MAET scanner, get first MAET images

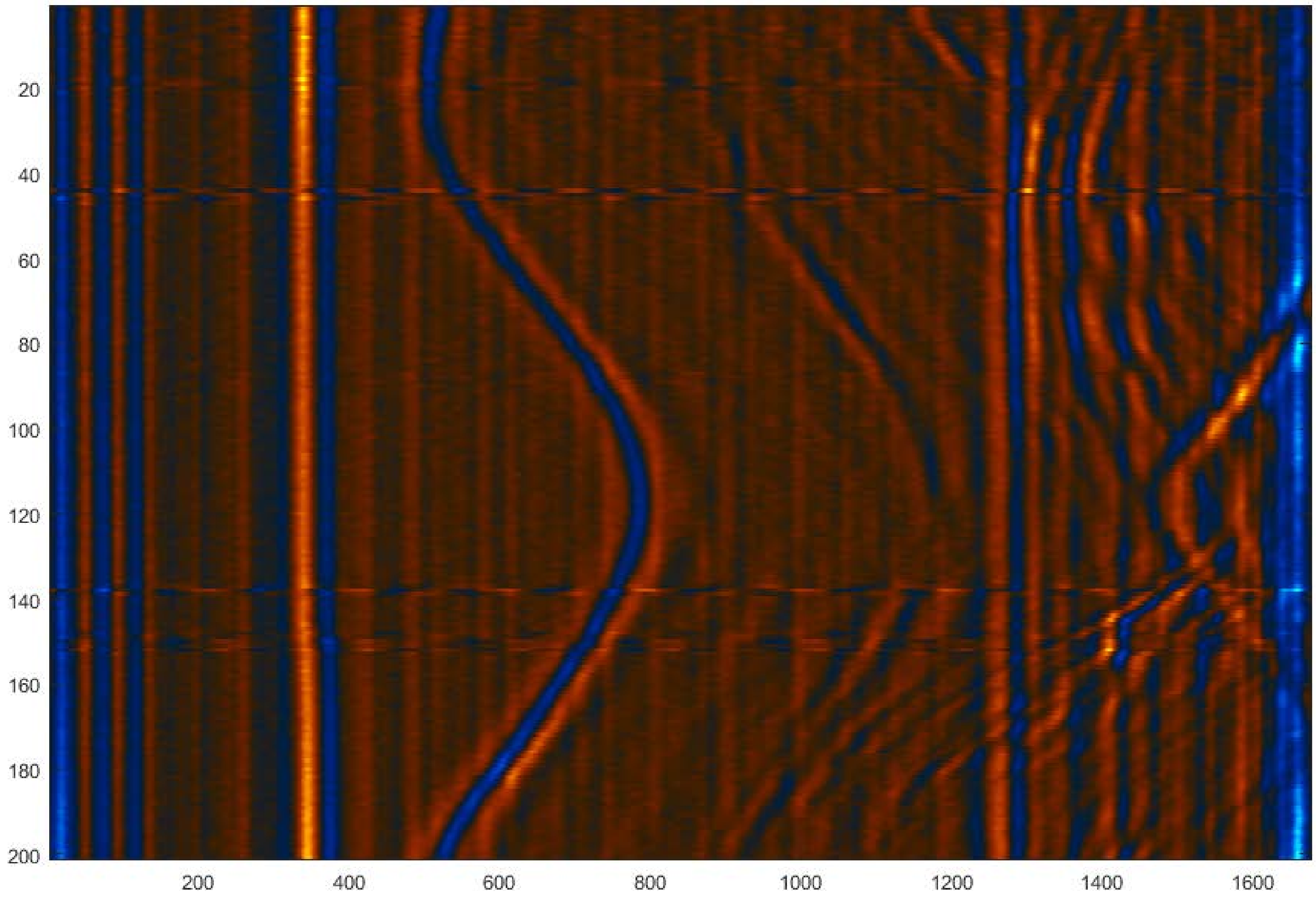
Parts were designed in SolidWorks and 3D-printed



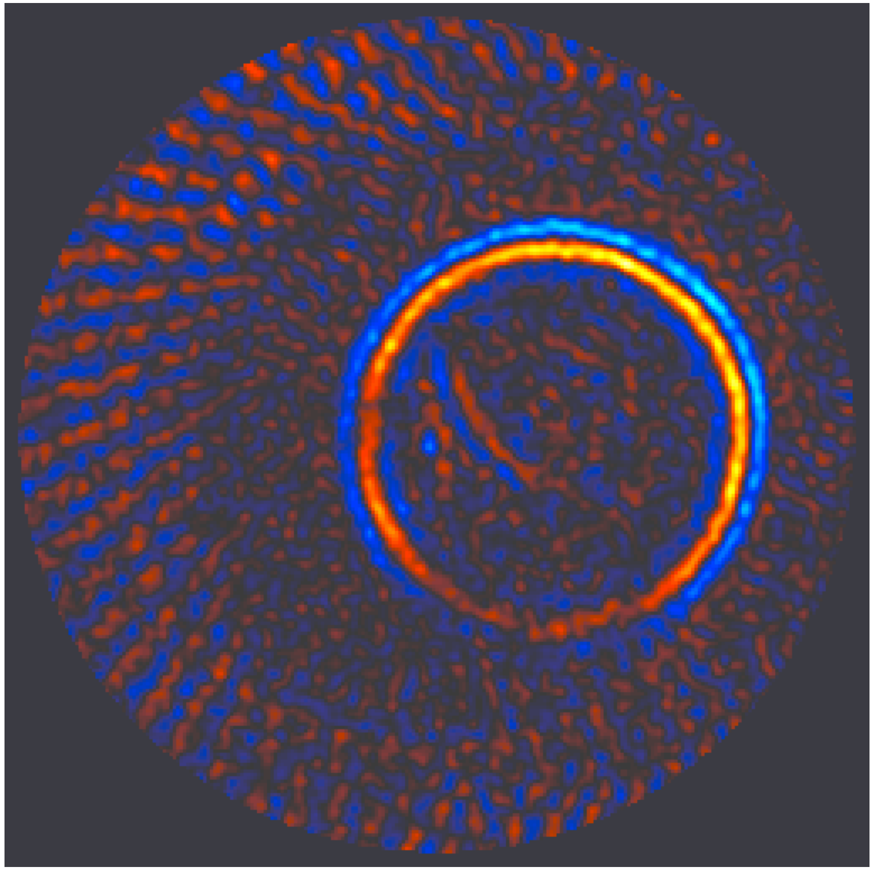
Fully assembled, in a tank, with a transducer

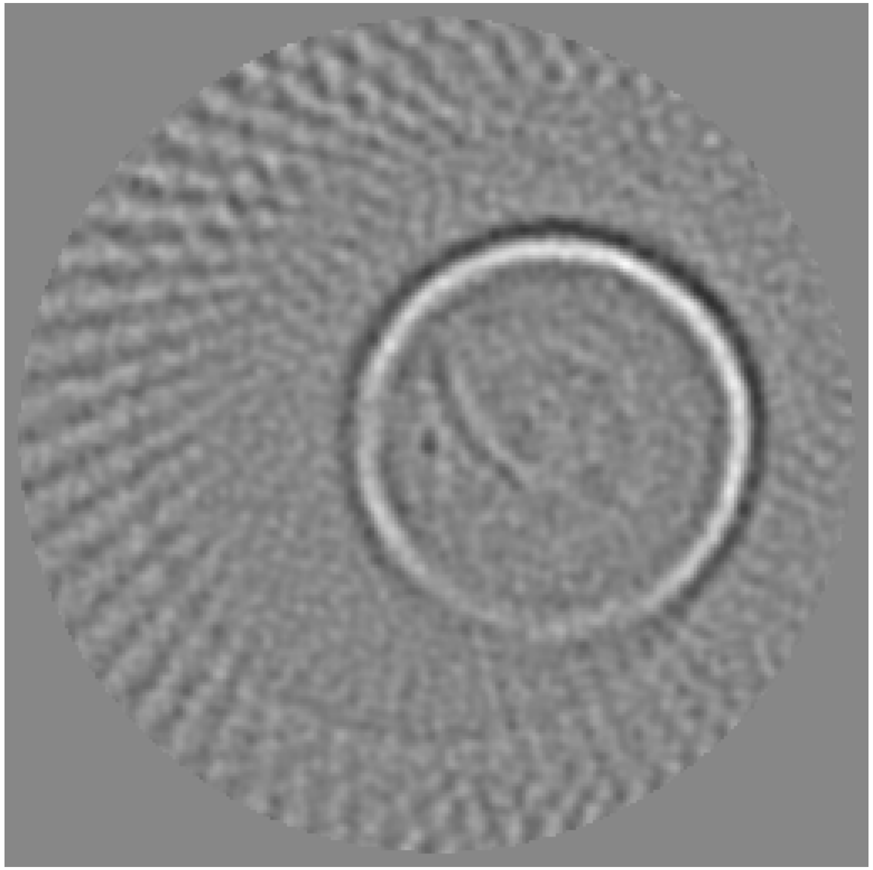


How does the signal look?

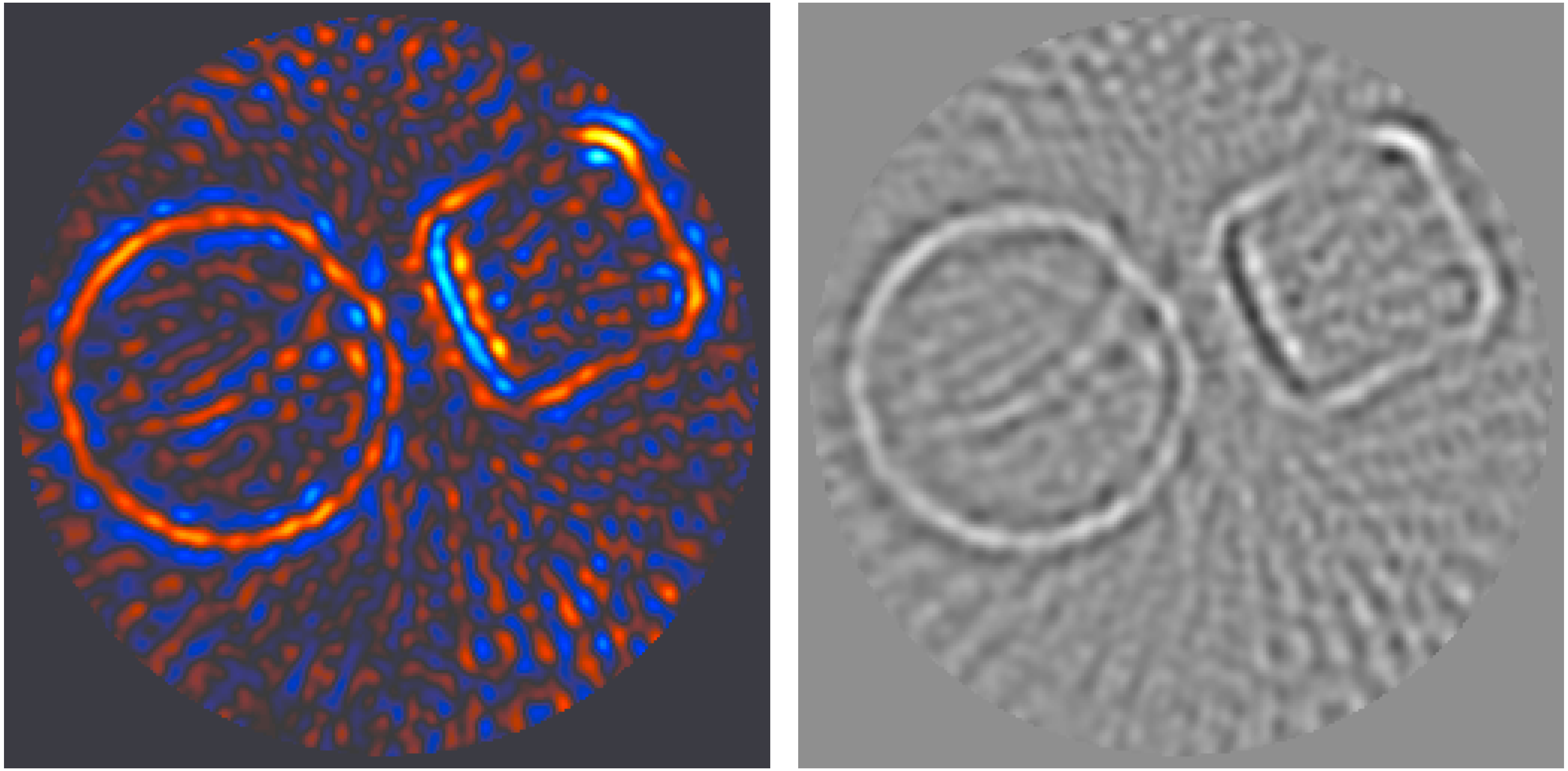


First reconstruction: round non-conducting phantom

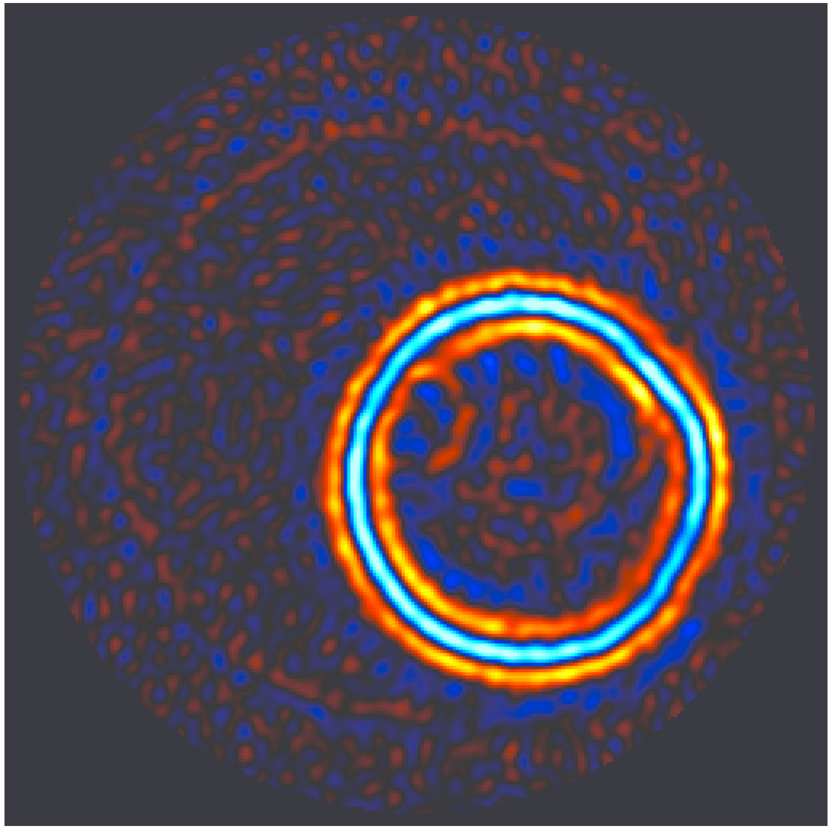


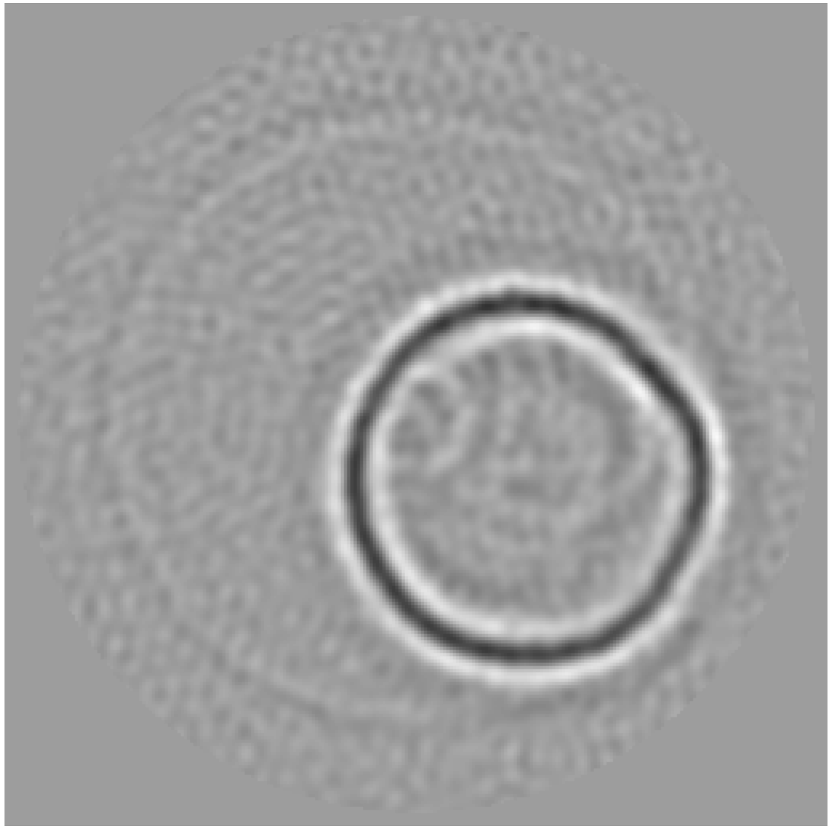


Round and square non-conducting phantoms

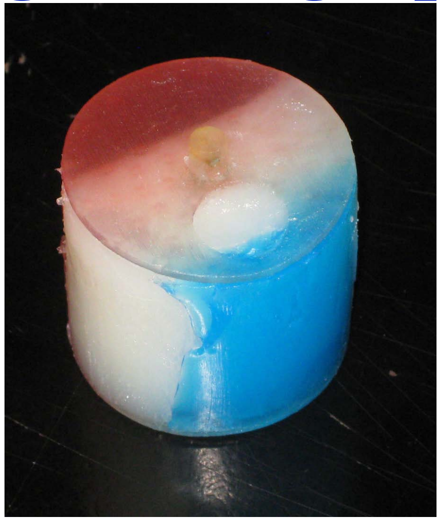


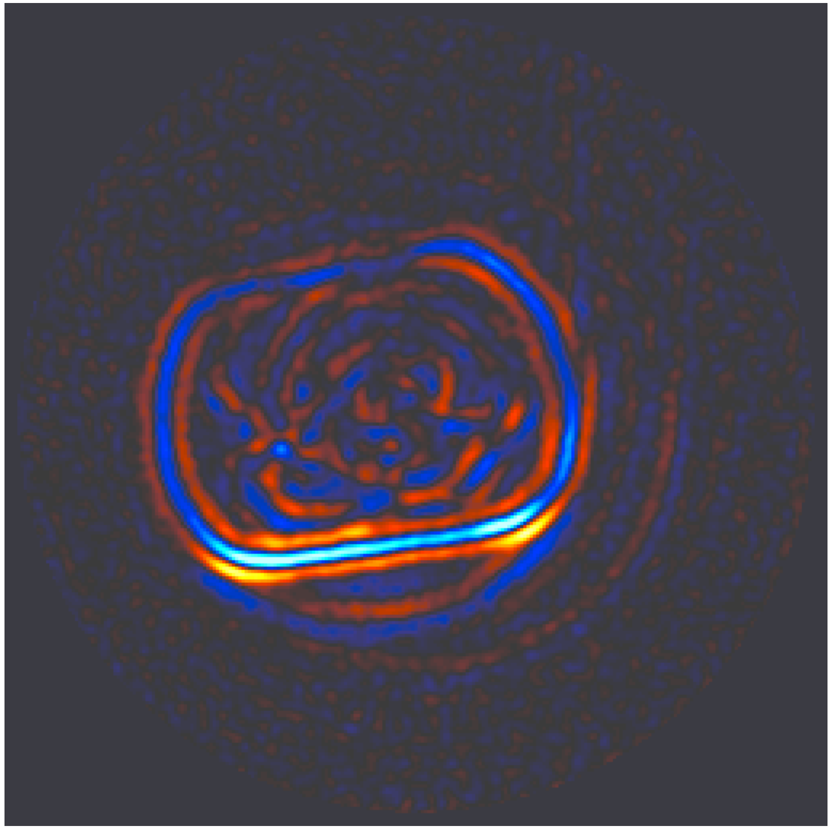
Round lard column, 30mm in diameter

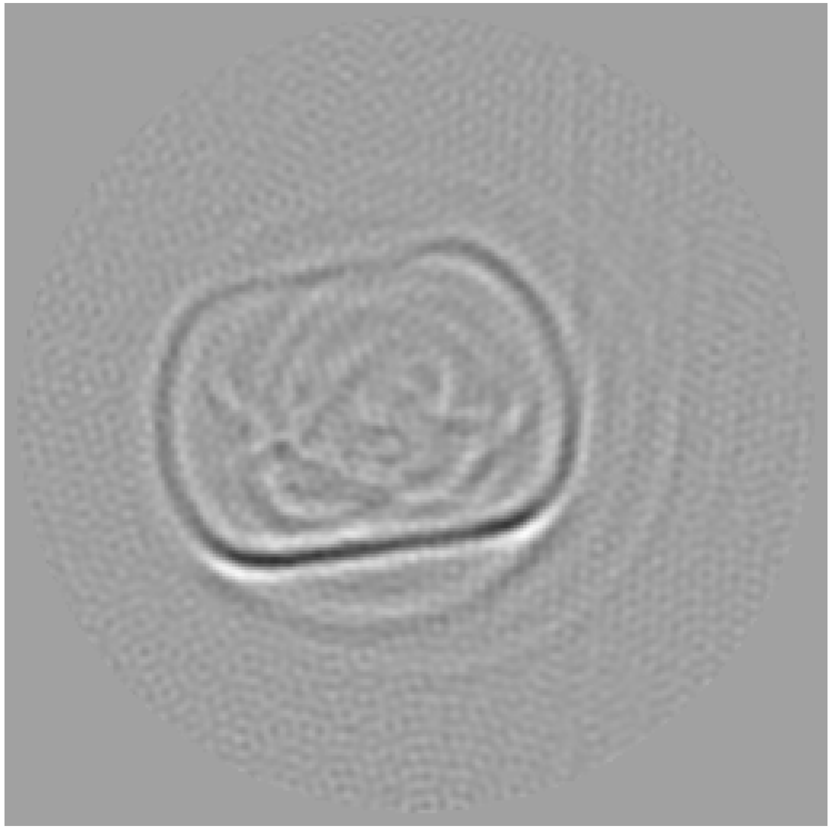




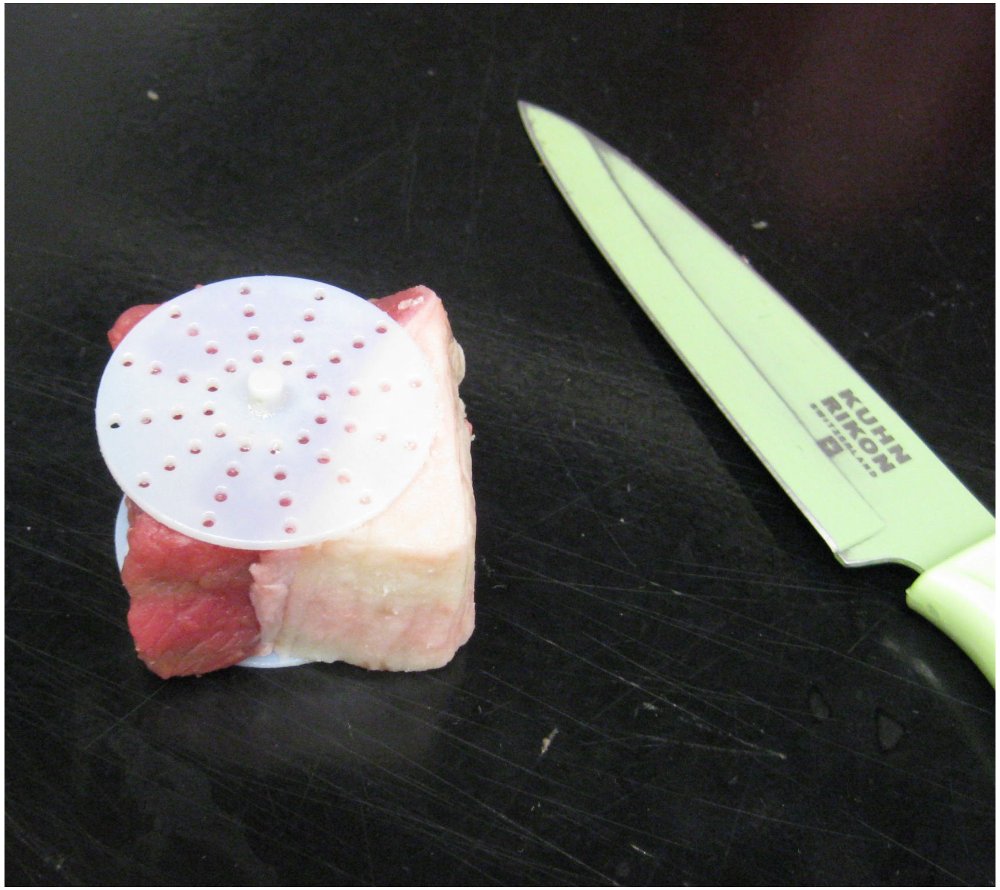
Layered gel-lard-gel phantom







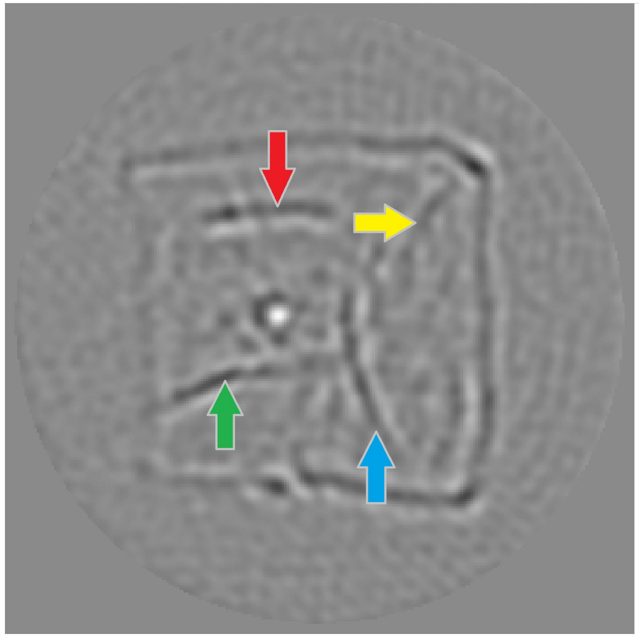
A beef sample



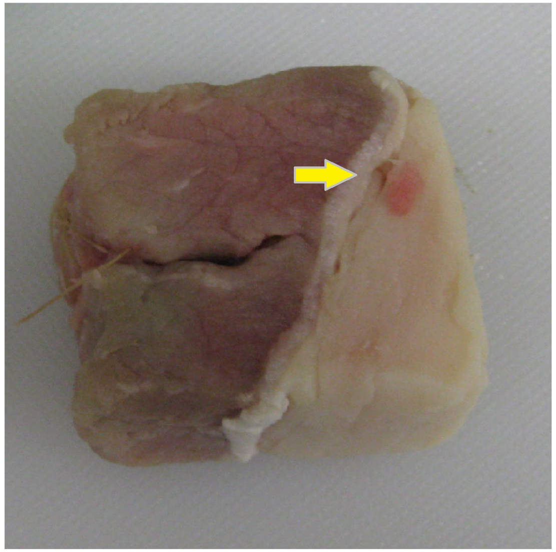
This is the first truly tomographic MAET image of a biological tissue.

Are the details in the image real?

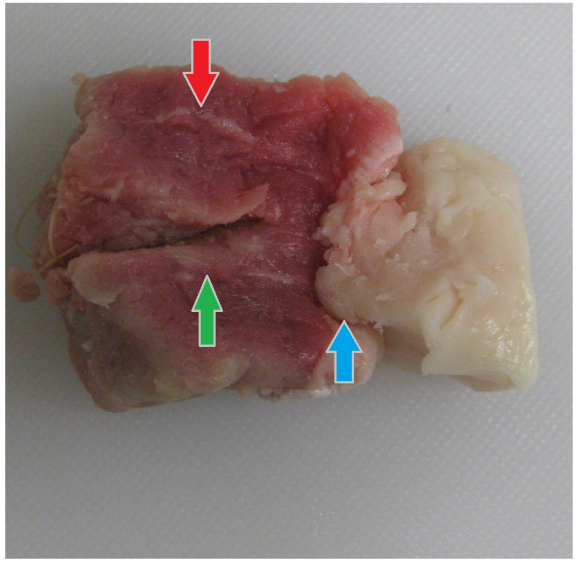
Image



Sample



Sample cut in half



What's next?

Photoacoustic generation of ultrasound waves?

MAET in a bore of an MRI scanner?

The end