100 Years of the Radon Transform

Linz, March 27-31, 2017

Johann Radon (1887-1956)

ÖAW RICAM

Welcome

to Linz, Austria, and thank you very much for attending the Conference *100 Years of the Radon Transform*, organized by the Johann Radon Institute for Computational and Applied Mathematics (RICAM), taking place from March, 27th to March, 31st, 2017 at the Johannes Kepler University of Linz (JKU).

The Johann Radon Institute of Computational and Applied Mathematics in Linz, Austria, celebrates the 100th anniversary of the publication of the famous paper "Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten" in Berichte über die Verhandlungen der Königlich-Sächsischen Gesellschaft der Wissenschaften zu Leipzig. Mathematisch-Physische Klasse, Band 69, 1917, Seiten 262 – 277, of the name giver of the RICAM institute, Johann Radon.

Johann Radon (1887-1956) was awarded a doctorate from the University of Vienna in Philosophy in 1910. After professorships in Hamburg, Greifswald, Erlangen, Breslau, and Innsbruck, he returned to the University of Vienna, where he became Dean and President of the University of Vienna. Johann Radon is well-known for his ground-breaking achievements in Mathematics, such as the Radon-transformation, the Radon-numbers, the Theorem of Radon, the Theorem of Radon-Nikodym and the Radon-Riesz Theorem. In the obituary of Johann Radon, published in Mathematischen Nachrichten, Bd. 62/3, 1958, by his colleague from the University of Vienna, Paul Funk, he outlined the above mentioned mathematical achievements but ignored the Radon transform. Today this might seem curious but from a pure mathematical point of view it is understandable. However after the realization of CT-scanners, which need some implementation of the inverse Radon transform, this must be reconsidered. Allan M. Cormack and Godfrey Hounsfield received in 1979 the Nobel-prize in Physiology or Medicine for the development of the first medical CT-scanner. It is an irony that both of them developed the algorithms for image reconstruction, independently from each other and from Radon. Allan M. Cormack studied the propagation of X-rays through human tissue and Godfrey Hounsfield was an engineer who was designing scanners.

Today Mathematics is established in many applications and permeates applications of biomedical engineering and of personalized Medicine. The appreciation is documented in an article of the newsletter of the Austrian Chamber of Pharmacists. This article was published on the occasion of the 50th day of death of Johann Radon. There Mag. Pharm. Franz Biba explained the development of X-ray CTs and the importance of Radon's work. The article ends with the statement that it is well-known that for Mathematics there is no nobel prize. Luckily there are similar prices. Recently, one of our participants received an outstanding award: Alexander Katsevich (University of Central Florida) was awarded the Marcus Wallenberg Prize by His Majesty the King of Sweden Carl Gustaf. Congratulations!

Recently, an article mentioning the conference has been published in the Austrian newspaper "Der Standard".

We sincerely hope that you enjoy your stay in Linz!

Local Organizing Committee

José A. Iglesias, RICAM (Co-Coordinator)
Kamran Sadiq, RICAM (Co-Coordinator)
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SITZUNG VOM 30. APRIL 1917.

Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten.

Von

JOHANN RADON.

Integriert man eine geeigneten Regularitätsbedingungen unterworfene Funktion zweier Veränderlichen x, y — eine *Punktfunktion* $f(P)$ in der Ebene — längs einer beliebigen Geraden g, so erhält man in den Integralwerten $F(g)$ eine *Geradenfunktion*. Das in Abschnitt A vorliegender Abhandlung gelöste Problem ist die Umkehrung dieser linearen Funktionaltransformation, d. h. es werden folgende Fragen beantwortet: kann jede, geeigneten Regularitätsbedingungen genügende Geradenfunktion auf diese Weise entstanden gedacht werden? Wenn ja, ist dann f durch F eindeutig bestimmt und wie kann es ermittelt werden?

Im Abschnitte B gelangt das dazu in gewisser Hinsicht duale Problem der Bestimmung einer Geradenfunktion $F(g)$ aus ihren Punktmittelwerten $f(P)$ zur Lösung.

Schließlich werden im Abschnitte C gewisse Verallgemeinerungen kurz besprochen, zu denen insbesondere die Betrachtung nichteuklidischer Mannigfaltigkeiten sowie höherer Räume Anlaß gibt.

Die Behandlung dieser an sich interessanten Probleme gewinnt ein erhöhtes Interesse durch die zahlreichen Beziehungen, die zwischen diesem Gegenstande und der Theorie des logarithmischen und NEWTONschen Potentiales bestehen, auf die an den bezüglichen Stellen zu verweisen sein wird.

A. Bestimmung einer Punktfunktion in der Ebene aus ihren geradlinigen Integralwerten.

1. Es sei $f(x, y)$ eine für alle reellen Punkte $P = [x, y]$ erklärte reelle Funktion, die folgende Regularitätsbedingungen erfülle:

a_1) $f(x, y)$ sei stetig.

b_1) Es konvergiere das über die ganze Ebene zu erstreckende Doppelintegral

$$\iint \frac{|f(x,y)|}{\sqrt{x^2+y^2}}dx\,dy.$$

c_1) Wird für einen beliebigen Punkt $P = [x, y]$ und jedes $r \geqq 0$

$$\bar f_P(r) = \frac{1}{2\pi}\int_0^{2\pi} f(x + r\cos\varphi,\ y + r\sin\varphi)\,d\varphi$$

gesetzt, so gelte für jeden Punkt P:

$$\lim_{r\to\infty} \bar f_P(r) = 0.$$

Dann gelten folgende Sätze:

Satz I: Der geradlinige Integralwert von f für die Gerade g mit der Gleichung $x\cos\varphi + y\sin\varphi = p$, der durch

$$\text{(I)}\quad F(p,\varphi) = F(-p, \varphi+\pi) = \int_{-\infty}^{+\infty} f(p\cos\varphi - s\sin\varphi,\ p\sin\varphi + s\cos\varphi)\,ds$$

gegeben ist, ist „im allgemeinen“ vorhanden; das soll heißen: auf jedem Kreise bilden die Berührungspunkte jener Tangenten, für welche F nicht existiert, eine Menge vom linearen Maße Null:

Satz II: Bildet man den Mittelwert von $F(p, \varphi)$ für die Tangenten des Kreises mit dem Zentrum $P = [x, y]$ und dem Radius q:

$$\text{(II)}\qquad \bar F_P(q) = \frac{1}{2\pi}\int_0^{2\pi} F(x\cos\varphi + y\sin\varphi + q,\ \varphi)\,d\varphi,$$

so konvergiert dieses Integral für alle P, q absolut.

Satz III: Der Wert von f ist durch F eindeutig bestimmt und läßt sich folgendermaßen berechnen:

$$\text{(III)}\qquad f(P) = -\frac{1}{\pi}\int_0^\infty \frac{d\bar F_P(q)}{q}.$$

Dabei ist das Integral im STIELTJESschen Sinne zu verstehen und kann auch durch die Formel:

$$\text{(III')}\qquad f(P) = \frac{1}{\pi}\lim_{\varepsilon\to 0}\left(\frac{\bar F_P(\varepsilon)}{\varepsilon} - \int_\varepsilon^\infty \frac{\bar F_P(q)}{q^2}\,dq\right)$$

definiert werden.

Indem wir zum Beweise schreiten, bemerken wir vorweg, daß die Bedingungen $a_1 - c_1$ gegenüber Bewegungen der Ebene invariant sind. Wir können also den Punkt $[0, 0]$ immer als Repräsentanten irgendeines Punktes der Ebene betrachten.

Man erkennt nun das Doppelintegral:

$$(1)\qquad \iint_{x^2+y^2>q^2} \frac{f(x,y)}{\sqrt{x^2+y^2-q^2}}\,dx\,dy$$

als absolut konvergent. Vermöge der Transformation

$$x = q\cos\varphi - s\sin\varphi,\quad y = q\sin\varphi + s\cos\varphi$$

geht dasselbe über in:

$$\int_0^{2\pi} d\varphi \int_0^\infty f(q\cos\varphi - s\sin\varphi,\ q\sin\varphi + s\cos\varphi)\,ds$$

$$= \int_0^{2\pi} d\varphi \int_{-\infty}^0 f(q\cos\varphi - s\sin\varphi,\ q\sin\varphi + s\cos\varphi)\,ds,$$

so daß man seinen Wert auch durch

$$\frac{1}{2}\int_0^{2\pi} d\varphi \int_{-\infty}^{+\infty} f(q\cos\varphi - s\sin\varphi,\ q\sin\varphi + s\cos\varphi)\,ds = \frac{1}{2}\int_0^{2\pi} F(q,\varphi)\,d\varphi$$

ausdrücken kann. Nach bekannten Eigenschaften der absolut konvergenten Doppelintegrale folgen hieraus die Behauptungen der Sätze I und II.

Um auf die Formel (III) zu kommen, kann man folgenden Weg einschlagen: Einführung von Polarkoordinaten in (1) liefert

$$\int_q^\infty dr \int_0^{2\pi} \frac{f(r\cos\varphi,\ r\sin\varphi)}{\sqrt{r^2-q^2}}\,d\varphi$$

Contents

Information

Conference Information

Registration. The conference registration will be on March 27th, 2017 from 8:30 - 9:30 am, on the 2nd floor of the University Center (see Campus plan and Conference venues).

Conference rooms. The conference will take place in the conference rooms UC 202G and UC 202DH, on the 2nd floor of the University Center and seminar room SP2 416 on the 4th floor of the Science Park Building 2. (see Campus plan and Conference venues).

Coffee breaks. The coffee breaks will be in the room adjacent to the conference rooms on the 2nd floor of the University Center.

Poster Session. Tuesday, March 28th, 3:45 pm - 4.15 pm, & Thursday, March 30th, 3:45 pm - 4.15 pm, on the 2nd Floor of University Center during the coffee break.

Conference Administrator.
Annette Weihs (RICAM Office - SP2 458), `radon100@ricam.oeaw.ac.at`, +43 732 2468 5233

Audiovisual & Computer Support.
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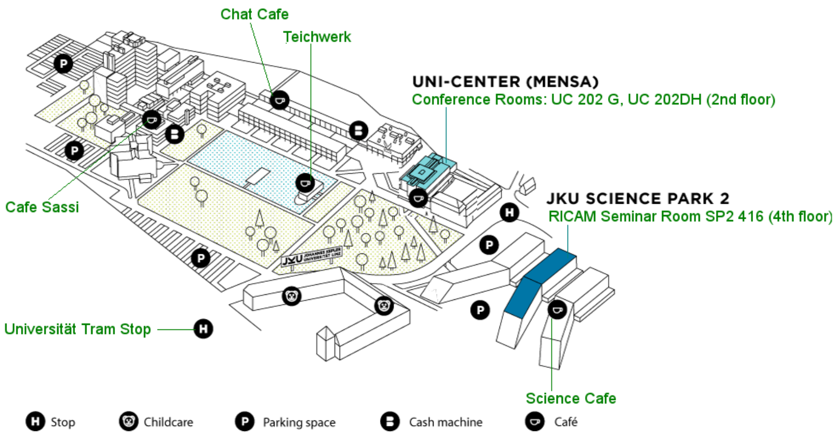


Campus plan and Conference venues

Restaurants and Cafés

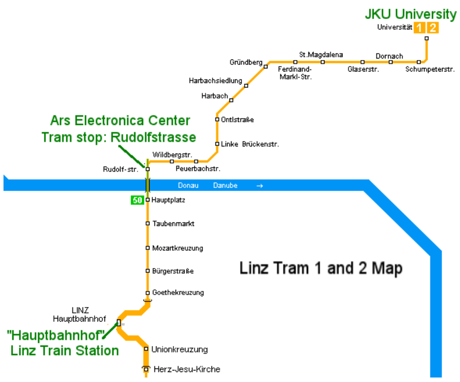
- Mensa Markt (lunch time only) - Main canteen of the University
- Teichwerk - restaurant (located at JKU pond)
  Schedule: Mon to Fri from 9:00 AM - midnight, Sat from 10:00 AM to midnight, and Sun from 10:00 AM to 6:00 PM
- Pizzeria “Bella Casa” - Italian and Greek restaurant (located next to the tram stop)
- Chinese restaurant “Jadegarten” - (located close by the tram stop, adjacent to “Bella Casa”)
- “Chat” café - coffee, drinks and sandwiches
  Schedule: Mon to Thur from 8:00 AM to 7:00 PM and Fri from 8:00 AM to 2:00 PM
- Café “Sassi” - coffee, drinks and small snacks (located in the building “Johannes Kepler Universität”) Schedule: Mon to Fri from 8:00 AM - 8:00 PM, Sat from 9:00 AM to 2:00 PM

- Science Café - coffee, drinks and sandwiches (located in the Science Park Building 3)
  Schedule: Mon to Thurs from 8:00 AM to 4:00 PM, Fri from 8:00 AM to 2:00 PM
- Bakery “Kandur” - bakery and small café (located opposite the tram stop)
- Burgerista - Hamburger Restaurant located at Altenbergerstraß6, 4040 Linz

Excursion Information

Ars Electronica: guided highlights tour
The Highlights Tour provides an overview of the Ars Electronica Centers top attractions. An expert tour guide accompanies you through all exhibition areas to experience New Views of Humankind.
See also the Ars Electronic webpage `http://www.aec.at/news/en/` for additional information.

Arriving by public transportation: Take the tram number 1 or 2 from the stop “Universität” to the stop Rudolfstraße (15 min ride). Please purchase a MAXI ticket at the tram stop (Euro 4,40; valid 24 hours).From there its just a short walk to the Ars Electronica Center; use the pedestrian underpass.



General Information

Orientation/ Local Transport. From the Linz railway station (“Hauptbahnhof”) you have to take tram number 1 or 2 in direction “Universität”. It takes about 25 minutes to reach the desired end stop “Universität”.

In order to get to the city center of Linz (“Hauptplatz”) and back you have to take again tram number 1 or 2 (about 20 minutes). For more information see `www.ricam.oeaw.ac.at/location/`.

Taxi Numbers.

+43 732 6969	Oberösterreichische Taxigenossenschaft
+43 732 2244	2244 Linzer Taxi
+43 732 781463	Enzendorfer Taxi & Transport
+43 732 2214	Linzer Taxi
+43 732 660217	LINTAX TaxibetriebsgesmbH

Further important phone numbers.

+43 (0)732 2457-0	Reception of Hotel Sommerhaus
133	General emergency number for the police
144	General emergency number for the ambulance

More information about RICAM can be found at `www.ricam.oeaw.ac.at`. See also the conference webpage `https://www.ricam.oeaw.ac.at/events/conferences/radon100/` for additional information.

Program

Monday, March 27th

08:30 - 09:30	Registration Location: UC 201 (2nd floor, University Center)
09:30 - 10:00	Opening: **Karl Kunisch, RICAM (Managing Scientific Director) & University of Graz** **Meinhard Lukas, Johannes Kepler University Linz (Rector)** **Heinz Engl, RICAM (Founding Director) & University of Vienna (Rector)**
10:00 - 10:45	Plenary Talk: Karl Sigmund **Johann Radon (1887-1956)** Location: UC 202 G (2nd floor, University Center)
10:45 - 11:00	Coffee Break
11:00 - 11:45	Plenary Talk: Gregory Beylkin **Radon transform and functions bandlimited in a disk** Location: UC 202 G (2nd floor, University Center)
11:45 - 13:30	Lunch Break
13:30 - 15:30	Minisymposia in parallel sessions: **MS 01: Atmospheric tomography in adaptive optics** Organizers: Tapio Helin and Daniela Saxenhuber Location: UC 202 DH (2nd floor, University Center) **MS 02: Discrete Tomograph** Organizer: Andreas Alpers Location: UC 202 G (2nd floor, University Center) **MS 03: Recent Developments on Inverse Scattering Problems** Organizer: Gang Bao Location: SP2 416 (4th floor, Science Park Building 2)
15:45 - 16:30	Coffee Break
16:30 - 18:30	Minisymposia in parallel sessions: **MS 04: Tomographic Reconstruction of Discontinuous Coefficients** Organizer: Elena Beretta Location: UC 202 DH (2nd floor, University Center) **MS 05: Analytic and Numerical Aspects of Radon Transforms** Organizers: Todd Quinto and Peter Kuchment Location: UC 202 G (2nd floor, University Center) **MS 06: Inverse problems in optical imaging** Organizer: John C. Schotland Location: SP2 416 (4th floor, Science Park Building 2)

Tuesday, March 28th

09:00 - 09:45	Plenary Talk: Michel Defrise **Simultaneous estimation of attenuation and activity in positron tomography** Location: UC 202 G (2nd floor, University Center)
09:45 - 10:00	Coffee Break
10:00 - 10:45	Plenary Talk: Christine De Mol **Nonnegative Matrix Factorization and Applications** Location: UC 202 G (2nd floor, University Center)
10:45 - 11:00	Coffee Break
11:00 - 11:45	Plenary Talk: Gabor Herman **Superiorized Inversion of the Radon Transform** Location: UC 202 G (2nd floor, University Center)
11:45 - 13:30	Lunch Break
13:30 - 15:30	Minisymposium: **MS 07: Analytic Aspects of Radon Transforms** Organizers: Todd Quinto and Peter Kuchment Location: UC 202 G (2nd floor, University Center)
15:45 - 16:30	**Coffee Break & Poster Presentation (2nd floor, University Center)**
16:30 - 18:30	Minisymposia in parallel sessions: **MS 08: Linear and non-linear tomography in non Euclidean geometries** Organizers: Plamen Stefanov, François Monard, and Gunther Uhlmann Location: UC 202 DH (2nd floor, University Center) **MS 09: Cone/Compton transforms and their applications** Organizers: Gaik Ambartsoumian and Fatma Terzioglu Location: UC 202 G (2nd floor, University Center) **MS 10: Vector and tensor tomography: advances in theory and applications** Organizer: Thomas Schuster Location: SP2 416 (4th floor, Science Park Building 2)

Wednesday, March 29th

09:00 - 09:45	Plenary Talk: Roman Novikov **Non-abelian Radon transform and its applications** Location: UC 202 G (2nd floor, University Center)
09:45 - 10:00	Coffee Break
10:00 - 10:45	Plenary Talk: Gaik Ambartsoumian **The broken-ray transform and its generalizations** Location: UC 202 G (2nd floor, University Center)
10:45 - 11:00	Coffee Break
11:00 - 11:45	Plenary Talk: Gunther Uhlmann **Travel time tomography and generalized Radon transforms** Location: UC 202 G (2nd floor, University Center)
11:45 - 13:30	Lunch Break
14:40	Meeting point “Ars Electronica” near tram station stop Rudolfstraße
15:00	Ars Electronica: guided highlights tour

Thursday, March 30th

09:00 - 09:45	Plenary Talk: Alexander Katsevich **Reconstruction algorithms for a class of restricted ray transforms without added singularities** Location: UC 202 G (2nd floor, University Center)
09:45 - 10:00	Coffee Break
10:00 - 10:45	Plenary Talk: Victor Palamodov **New reconstructions from the Compton camera data** Location: UC 202 G (2nd floor, University Center)
10:45 - 11:00	Coffee Break
11:00 - 11:45	Plenary Talk: Guillaume Bal **Stability estimates in inverse transport theory** Location: UC 202 G (2nd floor, University Center)
11:45 - 13:30	Lunch Break
13:30 - 15:30	Minisymposium: **MS 11: Applications of the Radon Transform** Organizer: Simon Arridge Location: UC 202 G (2nd floor, University Center)
15:45 - 16:30	**Coffee Break & Poster Presentation (2nd floor, University Center)**
16:30 - 18:30	Minisymposia in parallel sessions: **MS 12: Numerical microlocal analysis** Organizers: Marta Betcke and Jürgen Frikel Location: UC 202 DH (2nd floor, University Center) **MS 13: Radon-type transforms: Basis for Emerging Imaging** Organizers: Bernadette Hahn and Gaël Rigaud Location: UC 202 G (2nd floor, University Center) **MS 14: Theory and numerical methods for inverse problems and tomography** Organizer: Michael V. Klibanov Location: SP2 416 (4th floor, Science Park Building 2)

Friday, March 31th

09:00 - 09:45	Plenary Talk: Frank Natterer **Wave equation imaging by the Kaczmarz method** Location: UC 202 G (2^{nd} floor, University Center)
09:45 - 10:00	Coffee Break
10:00 - 10:45	Plenary Talk: Leonid Kunyansky **Rotational Magneto-Acousto-Electric Tomography: Theory and Experiments** Location: UC 202 G (2^{nd} floor, University Center)
10:45 - 11:00	Coffee Break
11:00 - 11:45	Plenary Talk: Alfred K. Louis **Cone Beam Tomography - from Radon's Point of View** Location: UC 202 G (2^{nd} floor, University Center)
11:45 - 13:30	Lunch Break
13:30 - 15:30	Minisymposia in parallel sessions: **MS 15: Towards Robust Tomography** Organizer: Samuli Siltanen Location: UC 202 DH (2^{nd} floor, University Center) **MS 16: Beyond filtered backprojection: Radon inversion with a priori knowledge** Organizers: Martin Benning, Matthias J. Ehrhardt, and Carola Schönlieb Location: UC 202 G (2^{nd} floor, University Center) **MS 17: Inverse problems for Radiative Transfer Equation and Broken Ray Approximation** Organizers: Linh Nguyen and Markus Haltmeier Location: SP2 416 (4^{th} floor, Science Park Building 2)
16:00 - 16:30	Closing

Plenary Talks

Johann Radon (1887-1956)

Karl Sigmund
Faculty for Mathematics, University of Vienna,
Oskar-Morgenstern-Platz 1, A-1090 Vienna, Austria

Radon transform and functions bandlimited in a disk

Gregory Beylkin University of Colorado at Boulder,
526 UCB, Boulder, CO 80309, USA

Abstract

David Slepian et. al. (circa 1960) introduced bases of eigenfunctions of space-and-band limiting operator (Prolate Spheroidal Wave Functions, PSWFs) in order to identify well-localized functions in both space and Fourier domains. Slepian also considered a multidimensional generalization, the eigenfunctions of the map of a disk (ball) in space to a disk (ball) in the Fourier domain. It turns out that the eigenfunctions of the map of a square (cube) in space to a disk (ball) in the Fourier domain is a more useful multidimensional generalization of PSWFs due to the properties of the spectrum. With the development of quadratures for bandlimited functions, it became possible to construct a Fast Fourier transform from a rectangular grid in space to a polar grid in a disk in the Fourier domain. As a result, the projection-slice theorem now has an accurate numerical implementation, thus yielding a fast algorithm for the Radon transform, dubbed the Polar Quadrature Inversion (PQI) algorithm. In addition, a rational signal model for projection data yields a method to augment the measured data, e.g. double the number of available samples in each projection or, equivalently, extend the domain of their Fourier transform in order to improve resolution near sharp transitions in the image.

Simultaneous estimation of attenuation and activity in positron tomography

Michel Defrise
Nuclear Medicine, Vrije Universiteit Brussel,
Laarbeeklaan 103, B-1090, Brussels, Belgium

Abstract

The talk concerns the simultaneous estimation of activity and attenuation in positron emission tomography (PET), and is based on work with Johan Nuyts and Ahmadreza Rezaei (KULeuven, Belgium), Koen Salvo (Vrije Universiteit Brussel), Yusheng Li, Samuel Matej and Scott Metzler (UPenn), and Vladimir Panin, Harshali Bal and Michael Casey (Siemens Healthcare, Knoxville, TN).

Attenuation correction in PET is usually based on information obtained from transmission tomography (CT) or magnetic resonance imaging (MRI). When such a direct measurement of the attenuation coefficient of the tissues is unreliable due for instance to patient motion or to the difficulty to identify bone in MRI, an alternative consists in estimating both the attenuation image μ and the activity image (tracer concentration) λ from the emission PET data y.

I will first consider an analytic model with full sampling,

$$y = e^{-\mathcal{X}\mu}\, \mathcal{X}_\sigma \lambda + b \tag{1}$$

where μ and λ are represented by functions and b is a known background. Here $\mathcal{X}$ denotes the x-ray transform and $\mathcal{X}_\sigma$ is the time-of-flight (TOF) x-ray transform, with σ the standard deviation of a gaussian TOF profile.

In classical (non-TOF) PET, $\sigma = \infty$, the data do not allow identifying the activity, except if λ consists of a finite number of point sources (F. Natterer 1987). The non-uniqueness of the solution to (1) is usually illustrated by examples with 2D radial objects μ, λ. More general examples of non-identifiable pairs μ, λ and μ', λ' that yield the same attenuated PET data y will be given, and this issue will be further discussed based on a property of the interior problem of tomography.

In the TOF case ($\sigma < \infty$, in current scanners typically $\sigma \simeq 20$ mm) the data determine the activity λ up to a multiplicative constant. The proof of this property is based on the range condition for $\mathcal{X}_\sigma$ and will be presented for 3D TOF PET with the *histoimage* data parameterization proposed by S. Matej et al (2009).

In practice, one uses in PET discrete image and data representations, which better model the physics of data acquisition. In that case, μ and λ are non-negative vectors, $\mathcal{X}$ and $\mathcal{X}_\sigma$ are matrices, and the data vector y is a realization of a Poisson variable with expectation $E\,(y\,|\mu, \lambda)$ given by the RHS of (1). The log-likelihood $\mathcal{L}(y, \mu, \lambda)$ is not concave, and has in general local maxima. Nevertheless good results have been obtained using algorithms to maximize the log-likelihood with respect to (μ, λ) (MLAA, J. Nuyts 1999) or with respect to $(a = e^{-\mathcal{X}\mu}, \lambda)$ (MLACF, A. Rezaei et al 2014), with a bi-concave log-likelihood in the latter case. Results will be illustrated by a recent work for brain PET studies (H Bal et al, 2017).

These two algorithms, MLAA and MLACF, are derived by applying standard surrogates to the likelihood, alternately for fixed attenuation and for fixed activity. An alternative algorithm (sMLACF, K. Salvo 2016), which simultaneously updates μ and λ, uses the Expectation-Maximization concept with complete variables, which extend the variables used by J. Fessler et al for a related problem in 1993. The derivation and properties of this algorithm will be summarized.

References can be found in the recent review by Y. Berker and Y. Li (Medical Physics 43, p. 807, 2016).

Nonnegative Matrix Factorization and Applications

Christine De Mol

Department of Mathematics and ECARES, Université Libre de Bruxelles,
Campus Plaine CPI 217, Boulevard du Triomphe, 1050 Brussels, Belgium

Abstract

This talk will discuss some aspects of the problem of nonnegative matrix factorization (NMF), i.e. of the factorization of a matrix with nonnegative elements into a product of two such matrices. While exact factorization can be used as a rank-reduction method for high-dimensional data, it should be replaced in the case of noisy data by an approximate factorization formulated as the minimization of a discrepancy term reflecting the statistics of the noise, namely a least-squares criterion for Gaussian noise and a Kullback-Leibler divergence for Poisson noise. Various regularization penalties can be added to this criterion according to the available prior knowledge. To solve the corresponding biconvex optimization problem, several alternating minimization strategies have been proposed, including multiplicative update schemes which can be derived through a Majorization-Minimization (MM) approach. Several convergence results pertaining to the resulting algorithms will be discussed. These algorithms can be successfully applied to hyperspectral imaging and to blind deconvolution of (nonnegative) images, as demonstrated by results of numerical simulations. In addition, work in progress concerning a new application to dynamic positron tomography will be presented. This is joint work with Michel Defrise (Vrije Universiteit Brussel) and Loïc Lecharlier (Université Libre de Bruxelles).

Superiorized Inversion of the Radon Transform

Gabor T. Herman
Graduate Center, City University of New York

Abstract

We discuss the Radon transform in 2D. We present the series expansion approach, which is an alternative to approximating the inverse Radon Transform.

In this approach, pactical inversion of the Radon Transform often uses constrained optimization, with the constraints arising from the desire to produce a solution that is constraints-compatible, where the constraints are provided by measured samples of the Radon Transform. It is typically the case that a large number of solutions would be considered good enough from the point of view of being constraints-compatible. In such a case, an secondary criterion is introduced that helps us to distinguish the better constraints-compatible solutions.

The superiorization methodology is a recently-developed heuristic approach to constrained optimization. The underlying idea is that in many applications there exist computationally-efficient iterative algorithms that produce constraints-compatible solutions. Often the algorithm is perturbation resilient in the sense that, even if certain kinds of changes are made at the end of each iterative step, the algorithm still produces a constraints-compatible solution. This property is exploited by using such perturbations to steer the algorithm to a solution that is not only constraints-compatible, but is also desirable according to a specified secondary criterion. The approach is very general, it is applicable to many iterative procedures and secondary criteria.

Most importantly, superiorization is a totally automatic procedure that turns an iterative algorithm into its superiorized version.

In the talk the mathematical definitions associated with superiorization will be outlined and theorems regarding some of its basic properties will be stated. The parctical performance of superiorization (as compared to classical constrained optimization) will be demonstrated in the the context of two applications: X-ray Computerized Tomography (CT) and Positron Emission Tomogrpahy (PET).

Non-abelian Radon transform and its applications

R.G. Novikov
CNRS, Centre de Mathématiques Appliquées,
Ecole Polytechnique, 91128 Palaiseau, France

Abstract

Considerations of the non-abelian Radon transform were started in [MZ] in the framework of the theory of solitons in dimension 2+1. On the other hand, the problem of inversion of transforms of such a type arises in different tomographies, including emission tomographies and polarization tomographies. In this talk we give a short review of old and recent results on this subject, including local and global reconstruction algorithms going back to [MZ], [N1], [N2] for the non-linear case.

References

[MZ] S.V. Manakov, V.E. Zakharov, Three-dimensional model of relativistic-invariant field theory, integrable by inverse scattering transform, Lett. Math. Phys. 5 (1981), 247-253

[N1] R.G. Novikov, On determination of a gauge field on $\mathbb{R}^d$ from its non-abelian Radon transform along oriented straight lines, J. Inst.Math. Jussieu 1 (2002), 559-629

[N2] R.G. Novikov, On iterative reconstruction in the nonlinearized polarization tomography, Inverse Problems 25 (2009) 115010

The broken-ray transform and its generalizations

Gaik Ambartsoumian Department of Mathematics,
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Abstract

The broken-ray transform (BRT), also often called V-line transform, is a generalized Radon transform that integrates a function along V-shaped trajectories, which consist of two rays with a common vertex. The study of these transforms was originally triggered by a mathematical model of single-scattering optical tomography (SSOT) introduced in 2009 by L. Florescu, V.A. Markel, and J.C. Schotland. Since then many scientists have studied the properties of BRT in various setups related to SSOT and some other imaging application, as well as from purely integral-geometric point of view. While BRT has many features common to other generalized Radon transforms, it has certain characteristics that are quite unique and can be attributed to the existence of a ridge in the path of integration. The talk will describe various setups of BRT and its generalizations with relevant applications, discuss known and new results related to the transform, and formulate some open problems.

Travel time tomography and generalized Radon transforms

Gunther Uhlmann

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Abstract

We consider the inverse problem of determining the sound speed or index of refraction of a medium by measuring the travel times of waves going through the medium. This problem arises in global seismology in an attempt to determine the inner structure of the Earth by measuring travel times of seismic waves. It has also several applications in optics and medical imaging among others.

The problem can be recast as a geometric problem: Can one determine a Riemannian metric of a Riemannian manifold with boundary by measuring the distance function between boundary points? This is the boundary rigidity problem. We will also consider the problem of determining the metric from the scattering relation, the so-called lens rigidity problem. The linearization of these problems involve the integration of a tensor along geodesics, similar to the X-ray transform.

We will also describe some recent results, joint with Plamen Stefanov and Andras Vasy, on the partial data case, where you are making measurements on a subset of the boundary. No previous knowledge of Riemannian geometry will be assumed.

Reconstruction algorithms for a class of restricted ray transforms without added singularities

Alexander Katsevich

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Abstract

Let X and X^* denote a restricted curvilinear ray transform and a corresponding backprojection operator, respectively. Analysis of reconstruction from the data Xf is usually based on a study of the composition X^*DX, where D is some local operator. If X^* is chosen appropriately, then X^*DX is a Fourier Integral Operator with singular symbol. The singularity of the symbol leads to the appearance of artifacts, which can be as strong as the original (or, useful) singularities. In the talk we propose a similar approach, but make two changes. First, we replace D with a nonlocal operator $\tilde{D}$ that integrates Xf along a curve in the data space. The result $\tilde{D}Xf$ resembles the generalized Radon transform R of f. The function $\tilde{D}Xf$ is defined on pairs $(x_0, \Theta) \in U \times S^2$, where $U \subset \mathbb{R}^3$ is an open set containing the support of f, and S^2 is the unit sphere in $\mathbb{R}^3$. Second, we replace X^* with a backprojection operator R^* that integrates with respect to Θ over S^2. It turns out that if $\tilde{D}$ and R^* are appropriately selected, then the composition $R^*\tilde{D}X$ is an elliptic pseudodifferential operator of order zero with principal symbol 1. Thus, we obtain an approximate reconstruction formula that recovers all the singularities correctly and does not produce artifacts.

New reconstructions from the Compton camera data

Victor P. Palamodov
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Abstract

Analytic methods for localization of gamma sources by means of the Compton camera will be in the focus of the talk.

Stability estimates in inverse transport theory

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Abstract

Inverse transport concerns the reconstruction of absorption and scattering coefficients from boundary measurements modeled by an albedo operator. Uniqueness and stability of the reconstructions are well understood in several natural settings. Absorption coefficients are typically reconstructed by an inverse Radon (inverse X-ray) transform from the most singular (ballistic) component of the albedo operator, while scattering coefficients are uniquely determined by the second-most singular (single scattering) component of the albedo operator in spatial dimension greater than or equal to 3. Standard stability estimates then show that the reconstructions are stable in appropriate metrics when the albedo operator is measured in the L^1 sense.

We claim that such error estimates may not be very informative in several practical settings. For instance, an arbitrary small blurring at, or mis-alignment of, the detectors results in a useless $O(1)$ error on the albedo operator since the stability estimates predict an $O(1)$ error on the reconstruction of the coefficients independent of said blurring or mis-alignment.

This talk revisits the stability estimate problem in the setting of a more forgiving metric, which will be chosen as the 1-Wasserstein metric since it penalizes blurring or mis-alignment by an amount that is proportional to the width of the blurring kernel or the amount of mis-alignment. We also consider errors on the construction of the probing sources in the same Wasserstein sense. Such a construction may be applied to the case with pure absorption, i.e., to the standard setting of application of the Radon transform, as well as to the case with non-vanishing scattering coefficients. We obtain new stability estimates of Hölder type in this setting, which should more faithfully model practical reconstruction errors.

This is joint work with Alexandre Jollivet.

Wave equation imaging by the Kaczmarz method

Frank Natterer
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University of Münster, Germany

Abstract

The Kaczmarz method was one of the first methods to invert the Radon transform. In the last decades it has been extended to nonlinear imaging techniques, most notably to solving inverse problems based on the wave equation, such as ultrasound tomography and full waveform inversion in seismic imaging. We derive Kaczmarz type algorithms in analogy to the ART algorithm of X-ray tomography. We give heuristic conditions on the initial approximation for convergence and study in particular the case of missing low frequencies in the source pulse. Various methods for speeding up the convergence, such as plane wave stacking and source encoding, are discussed. Finally we indicate the changes which have to be made for media with memory.

Rotational Magneto-Acousto-Electric Tomography: Theory and Experiments

Leonid Kunyansky
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Abstract

Magneto-Acousto-Electric tomography is a novel imaging modality promising to overcome shortcomings of the Electrical Impedance tomography, and to deliver a high-resolution reconstruction of the electrical conductivity within the object of interest. It is based on measurements of the electric potential generated by the Lorentz force acting on free ions moving in a magnetic field. In the first part of my talk I will explain the underlying physics, and will present a general scheme of solution of the associated inverse problems. Next, I will present a realistic 2D version of this modality, with a rotating object of interest and a band limited transducer, that models the 2D MAET scanner we have actually built. I will discuss the new mathematical techniques supporting this device, such as the use of a synthetic flat transducer and synthetic rotating currents. Finally, the first images obtained using this scanner will be demonstrated and analyzed. (Joint work with R.S. Witte and C.P. Ingram)

Cone Beam Tomography - from Radon's Point of View

Alfred K LOUIS Saarland University

Abstract

The formula of Grangeat, relating cone beam and Radon transform is fundamental for the derivation of inversion formulas for the cone beam transform. It also can be used for establishing general inversion formulas similar to the Radon transform of filtered backprojection type. A small modification results in an inversion formula for the gradient of the searched-for quantity, which proves to be useful in feature and contour reconstruction going beyond the slice by slice approach occasionally applied. Recently Kazantsev presented a singular value decomposition for the cone beam transform for source positions on a sphere surrounding the object. He thus complemented the results of Maass for the parallel X-ray transform. Again Grangeat's formula can be used to tackle the problem of singular value decomposition for more realistic scanning geometries. Numerical experiments with the calculation of the gradient present the possibility of avoiding the detailed use of the scanning curve, which for discretely measured data has no influence on the data. For the circular scanning, mostly used in nondestructive testing, cracks orthogonal to the plane where the source is moved can be determined. For Feldkamp type of algorithms this quantity only can be attacked by differentiation form slice to slice.

Minisymposia

MS 01: Atmospheric tomography in adaptive optics

Organizers: Tapio Helin and Daniela Saxenhuber

On the compression of atmospheric layer profiles to fewer layers for tomographic reconstruction

Günter Auzinger Industrial Mathematics Institute,
Johannes Kepler University Linz, Austria

Abstract

In wide-field applications of adaptive optics systems, the problem of atmospheric tomography has to be solved: Given measurements of wave-front sensors gathering light from guide stars in several directions of view, the fluctuations of the refractive index (so-called phase delays) in turbulent layers of the atmosphere have to be calculated. The results are subsequently used for controlling deformable mirrors in order to compensate the wavefront aberrations caused by these phase-delays, thereby increasing the quality of the scientific images over the field of view.

Due to run-time restrictions and stability requirements on the tomographic solver, the number of layers on which the solver is operating, is in practice much smaller than the number of physical turbulent layers in the atmosphere. The problem of finding appropriate heights for these fewer reconstruction layers is referred to as layer compression. We show by means of numerical simulations that the choice of these reconstruction layers has a significant impact on the quality that can be reached by a solver. We give an overview on existing methods for layer compression and focus on the optimal grouping method, which seems to be the most promising approach at the current state of research.

Efficient tomographic wave-front reconstruction in astronomical adaptive optics exploiting Toeplitz matrix structure

Yoshito Ono Aix Marseille Université, CNRS,
LAM (Laboratoire d'Astrophysique de Marseille) UMR 7326,
13388 Marseille, France

Abstract

For Laser Tomographic Adaptive Optics (LTAO) systems in future Extreme Large Telescopes (ELTs), we need to estimate tomographically the three-dimensional structure of phase distortion due to the atmospheric turbulence above a telescope from slopes measured by multiple Wave-Front Sensors (WFSs) observing different Laser Guide Stars (LGSs). This tomographic estimation can be achieved by a Minimum-Mean-Square-Error (MMSE) algorithm using phase and slope covariance matrices, which is the main computation burden in a real-time control for future LTAO systems. In this presentation, an efficient MMSE tomographic estimation method using the Toeplitz structure in the covariance matrix is presented. We explain how to apply the Toeplitz-based method to the tomographic estimation with multiple LGS and also how to compute the theoretical covariance matrix with different models. We then compare it to sparse matrix techniques in terms of the estimation performance and the computation burden (memory and speed) by end-to-end numerical simulation assuming a future LTAO system on a 37 m diameter telescope. The off-line analytical estimation of the tomographic error with the Toeplitz-based method is also discussed.

This is a joint work with Carlos M. Correia.

Determining ground layer turbulence statistics using a SLODAR-type method

Jonatan Lehtonen University of Helsinki, Finland

Abstract

Adaptive optics systems are designed to improve the the imaging quality of ground based telescopes by providing real-time compensation for the unwanted optical aberrations generated by the atmospheric turbulence. A key component in these systems is a statistical inverse problem called atmospheric tomography, where the goal is to reconstruct the atmospheric turbulence above the telescope. Due to the extremely small angle of view (around 1 arcmin), this inverse problem relies on solid prior information. This leads to the independent problem of turbulence profiling, which aims to reconstruct the vertical turbulence strength profile of the atmosphere.

Spatial correlations from observations of two guide stars can be used to estimate the turbulence profile; this is the idea at the heart of SLODAR-type methods. These methods rely on the assumption that the turbulence statistics at any altitude can be accurately described by the Kolmogorov model. However, while this model is quite accurate in most of the atmosphere, it is well-known that the turbulence statistics deviate from the Kolmogorov model close to the ground, and this issue is emphasized by the fact that a significant part of the turbulence strength is often located very close to the ground. In this talk, we discuss the possibility of identifying non-Kolmogorov turbulence models close to the ground based on a SLODAR-type methods. This is joint work with Tapio Helin, Stefan Kindermann and Ronny Ramlau.

A Singular Value Type Decomposition for the Atmospheric Tomography Operator

Andreas Neubauer Industrial Mathematics Institute,
Johannes Kepler University Linz, A-4040 Linz, Austria

Abstract

The new generation of Extremely Large Telescopes currently under construction relies on Adaptive Optics techniques for the correction of image degradation due to atmospheric turbulences. A crucial part of the correction is related to Atmospheric Tomography, which will be analyzed in this paper. In particular, we derive a singular value type decomposition for the related operator. Despite being a limited angle problem, we show that Atmospheric Tomography is, in general, not ill-posed, although ill-conditioned.

This is joint work with Ronny Ramlau (from the same institute).

Turbulence tomography for Astronomical Adaptive Optics

Carlos M. Correia Aix Marseille Université, CNRS,
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Abstract

We provide an overview of requirements for tomographic systems on Extreme Large Telescopes (ELTs), iterative and non-iterative solvers devised to tackle the huge amount of real-time computations required. We then move to present the optimal dynamic solution in closed-loop operation involving Kalman filters. We tour the recent progress towards rendering Kalman filters suitable for driving astronomical adaptive optics with increasing numbers of degrees of freedom and discuss the prospects to port them to the foreseen real-time architectures. Illustrative examples are given for the instrument Harmoni on the 37m European-ELT.

This is a joint work with Yoshito Ono, Paolo Massioni, and Benoit Neichel.

Tomographic Reconstruction for the METIS Instrument

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Abstract

METIS is one of the three first light instruments for the European Extremely Large Telescope (E-ELT).

METIS will be operating in the mid-infrared, at wavelengths between 3 and 19 micrometer. Although the impact of the atmospheric turbulence is significantly less than for other instruments on the E-ELT, operating at shorter wavelengths, METIS will still rely heavily on its Adaptive Optics Systems. METIS will initially be installed with only a Single Conjugate Adaptive Optics System (SCAO). This SCAO system will only be able to provide correction for the brightest stars. In order to extend the sky coverage, a Laser Tomography Adaptive Optics system is foreseen that will allow for both correction on fainter stars as well as provide a more uniform performance over the field of view of METIS.

The METIS LTAO system will be operating with a single Deformable Mirror (DM), namely the M4(+M5) mirror of the E-ELT and is currently foreseen to use all 6 Laser Guide Stars (LGS) and up to 3 Natural Guide Stars (NGS) for sensing the atmospheric turbulence. METIS is severely constrained in the possible configurations for its LGS; due to the optical system of the telescope, the need to keep the scientific field clear and the large range in (de)focus of the LGS over the range of Zenith Angles, only a single asterism with a radius of 1.3 arcminutes is possible.

In this paper we will introduce the METIS instrument, the AO systems and operation. We will explain the design and constraints on its LTAO system and impact on the performance. Lastly, we will discuss the tomographic reconstruction, the limitations on the reconstruction and its performance

MS 02: Discrete Tomography
Organizer: Andreas Alpers

The Discrete Algebraic Reconstruction Technique (DART): successes, shortcomings, and prospects

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Abstract

The Discrete Algebraic Reconstruction Technique (DART) was one of the first discrete tomography algorithms capable of dealing with real-world experimental datasets in an effective way. While heuristic in nature and without theoretical guarantees on the quality of the resulting images, the DART algorithm turns out to be highly effective in a broad range of practical applications. After early successes in applying it to various types of electron tomography and X-ray tomography data, it also became clear that DART has several limitations. First of all, the algorithms has several parameters, and finding optimal values for these can be challenging in a real-world setting, requiring manual parameter tuning. Second, the strong assumption on the discreteness of the grey levels can cause artefacts if the measured image data does not satisfy the discrete model perfectly. Finally, the computational requirements of DART imposes practical hurdles on its use in a high-throughput environment. Recently, substantial progress has been made in developing new algorithms based on similar concepts as DART, that do not have these shortcomings, such as the recently proposed TVR-DART algorithm (joint work with Xiaodong Zhuge).

In this talk, the basic ideas behind the DART algorithm will be discussed, why it works effectively in cases that satisfy its key requirements, and why it fails in cases that go outside its applicability range. For each of the three key problems mentioned above, solution strategies will be discussed. The viability of using discrete tomography in a practical setting will be demonstrated for a series of challenging experimental datasets.

Three Problems in Discrete Tomography: Reconstruction, Uniqueness, and Stability

Sara Brunetti University of Siena,
Dipartimento di Ingegneria dell'Informazione e Scienze Matematiche,
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Abstract

When the available X-ray data is insufficient, the accuracy of the tomographic reconstruction is likely to be inadequate. The assumption that the densities of the object materials are restricted to few known grey level values is on the basis of the definition of Discrete Tomography and permits partially to deal with the accuracy problem. The objects studied here are lattice sets and their X-rays in a lattice direction count the number of their points lying on each line parallel to the given direction. In the literature, the reconstruction, and uniqueness issues have been intensively studied from different viewpoints. In the general setting, the problems are easy when the X-rays are taken from two directions, and become hard for more than two directions. Analogously negative results arise when the stability of the reconstruction task is studied.

A common way to deal with these problems is to incorporate a priori knowledge about the object. Therefore, in the literature special classes of geometric objects are considered such as convex, Q-convex and additive lattice sets. A different restriction, in the same spirit, consists in considering bounded sets, i.e., subsets of a given rectangular grid.

We review and discuss the three problems especially for Q-convex and bounded additive lattice sets. Finally we discuss some recent results, and perspectives.

Geometric tomography: X-ray transforms and uniqueness

Richard Gardner Western Washington University

Abstract

This talk will provide a very brief introduction to one aspect of geometric tomography, focusing on the determination of convex sets via the X-ray transform, divergent beam transform, or circular Radon transform of their characteristic functions. The emphasis will be on uniqueness, recalling some known results and stating interesting open problems.

On double-resolution imaging in discrete tomography

Peter Gritzmann Zentrum Mathematik, Technische Universität München,
D-85747 Garching bei München, Germany

Abstract

Super-resolution imaging aims at improving the resolution of an image by enhancing it with other images or data that might have been acquired using different imaging techniques or modalities. We consider the task of doubling the resolution of tomographic grayscale images of binary objects by fusion with double-resolution tomographic data that has been acquired from two viewing angles. We show that this task is polynomial-time solvable if the gray levels have been reliably determined. The task becomes $\mathbb{NP}$-hard if the gray levels of some pixels come with an error of ± 1 or larger. The $\mathbb{NP}$-hardness persists for any larger resolution enhancement factor. This means that noise does not only affect the quality of a reconstructed image but, less expectedly, also the algorithmic tractability of the inverse problem itself. (Joint work with Andreas Alpers, Munich)

Variational and Numerical Approaches to Large-Scale Discrete Tomography

Christoph Schnörr Institute of Applied Mathematics,
Heidelberg University, Germany

Abstract

The reconstruction of functions that take values in a finite set, from linear projection measurements, may be regarded as an image labeling problem where direct data observations are missing. Sparse gradient supports enable to estimate the number of projections required for unique recovery. The talk reports recent progress along these lines based on ideas that connect discrete tomography to the field of compressed sensing and to numerical methods of variational image analysis.

Joint work with: Stefania Petra, Heidelberg University.

Consistency conditions for discrete tomography

Rob Tijdeman Mathematisch Institute, Leiden University,
P.O. Box 9512, 2300 RA Leiden, The Netherlands

Abstract

This is joint work with Lajos Hajdu (Debrecen). For continuous tomography Helgason and Ludwig developed consistency conditions. They were used by others to overcome defects in measurements. In this paper we introduce a consistency criterion for discrete tomography. We indicate how the consistency conditions can be used to overcome defects in measurements.

MS 03: Recent Developments on Inverse Scattering Problems
Organizer: Gang Bao

Forward and inverse scattering problems for multi-particle system via FMM and NUFFT

Jun Lai School of Mathematical Sciences, Zhejiang University,

Abstract

Mutli-particle scattering arises from several important applications including biomedical optics, atmospheric radiation, remote sensing, etc. In this talk, we study the scattering from some extended obstacles within a large number of small particles. The forward scattering problem is formulated via generalized Foldy-Lax formulation. Fast Multiple Method(FMM) is applied to accelerate the evaluation of the far field pattern. For the inverse problem, a direct imaging method is proposed with the indicator function evaluated through non-uniform FFT(NUFFT). Numerical experiments are presented to show the effectiveness and efficiency of the algorithm.

Stability in inverse source problems for wave propagation

Peijun Li Department of Mathematics, Purdue University

Abstract

This talk concerns the stability in the inverse source problems for acoustic, elastic, and electromagnetic waves. We show that the increasing stability can be achieved by using the Dirichlet boundary data only at multiple frequencies.

Increasing stability in the inverse source problem with attenuation and many frequencies

Shuai Lu School of Mathematical Sciences, Fudan University, Shanghai China

Abstract

We study the interior inverse source problem for the Helmholtz equation from boundary Cauchy data of multiple wave numbers. The main goal of this paper is to understand the dependence of increasing stability on the attenuation, both analytically and numerically. To implement it we use the Fourier transform with respect to the wave numbers, explicit bounds for analytic continuation, and observability bounds for the wave equation. In particular, by using Carleman estimates for the wave equation we trace the dependence of exact observability bounds on the constant damping. Numerical examples in 3 spatial dimension support the theoretical results. It is a joint work with Prof. Victor Isakov (Wichita State University).

Towards the identification of compact perturbations of periodic layers – preliminary results.

Andreas Kirsch Department of Mathematics,
Karlsruhe Institute of Technology (KIT),
76131 Karlsruhe, Germany

Abstract

This project is joint work with Armin Lechleiter (University of Bremen, Germany). The ultimate goal of this project is to determine a compact perturbation of a known layer from field measurements of the scattered fields produced by some incident field. The layer of given height h is decribed by an index of refraction $n(x) = n(x_1, x_2)$ for $x = (x_1, x_2) \in \mathbb{R} \times \mathbb{R}_{x_2>0}$ which is periodic respect to x_1 and constant for $x_2 \geq h$. Bevor treating the inverse problem the corresponding direct problem has to be investigated carefully. In this talk we concentrate on the direct scattering problem and show that the limiting absorption principle holds which provides a radiation condition. This radiation condition allows the existence of well-determined modes which propagate to the left and right. These results provide the proper settings for the spaces in order that the "parameter to solution map" is smooth.

Inverse Scattering Problems With Phaseless Far-field Data

Bo Zhang Academy of Mathematics and Systems Science, Chinese Academy of Sciences,
Beijing 100190, China and School of Mathematical Sciences,
University of Chinese Academy of Sciences, Beijing 100049, China

Abstract

In this talk, we give a brief review on uniqueness results and numerical methods for inverse scattering problems with phaseless far-field data, obtained recently in our group. These results include recursive Newton iteration methods for reconstructing acoustic obstacles from multi-frequency phaseless far-field data, direct imaging algorithms with phaseless far-field data at a fixed frequency, and unique determination of acoustic obstacles and inhomogeneous media from phaseless far-field data at a fixed frequency. This talk is based on joint works with Xiaoxu Xu and Haiwen Zhang.

MS 04: Tomographic Reconstruction of Discontinuous Coefficients
Organizer: Elena Beretta

Disjoint sparsity for signal separation and applications to quantitative photoacoustic tomography

Giovanni S. Alberti University of Genoa, Department of Mathematics

Abstract

This is joint work with H Ammari. The main focus of this talk is the reconstruction of the signals f and g_i, $i = 1, \ldots, N$, from the knowledge of their sums $h_i = f + g_i$, under the assumption that f and the g_is can be sparsely represented with respect to two different dictionaries A_f and A_g. This generalises the well-known "morphological component analysis" to a multi-measurement setting. The main result states that f and the g_is can be uniquely and stably reconstructed by finding sparse representations of h_i for every i with respect to the concatenated dictionary $[A_f, A_g]$, provided that enough incoherent measurements g_is are available. The incoherence is measured in terms of their mutual disjoint sparsity.

This method finds applications in the reconstruction procedures of several hybrid imaging inverse problems, where internal data are measured. These measurements usually consist of the main unknown multiplied by other unknown quantities, and so the disjoint sparsity approach can be directly applied. In this case, the feature that distinguishes the two parts is the different level of smoothness. As an example, I will show how to apply the method to the reconstruction in quantitative photoacoustic tomography, also in the case when the Grüneisen parameter, the optical absorption and the diffusion coefficient are all unknown.

A linear elastic model to detect magma chamber

Aspri Andrea Department of Mathematics, Sapienza - Universit di Roma

Abstract

I will present a physical model used in volcanology to describe grounds deformations within calderas. Based on the linear elastic theory, this analytical model replaces caldera with a homogeneous, isotropic half-space and the magma chamber by a pressurized cavity. I will exhibit results on the well-posedness of this problem within the framework of layer potentials techniques. Then, adding the hypothesis of small dimensions of the cavity with respect to its depth, the asymptotic analysis for the solution of the problem will be addressed. After that, I will show a stability estimate for the inverse problem of determining the pressurized cavity from a single measurement of the displacement taken on a portion of the boundary of the half-space.

Differentiability of the Dirichlet to Neumann map under movements of polygonal inclusions.

Elisa Francini Università di Firenze

Abstract

I will review some recent results concerning the differentiability of the Dirichlet to Neumann map under movements of polygonal inclusions for the Helmholtz equation or the conductivity equation. The formula for the derivative that we obtain can be used to prove stability results in the reconstruction of interfaces from boundary measurements and has also application in the context of shape optimization. (Joint work with Elena Beretta and Sergio Vessella)

Numerical approximation of Bayesian Inverse Problems for PDEs by Reduced-Order Modeling techniques

Andrea Manzoni CMCS-MATH-SB, Ecole Polytechnique Fédérale de Lausanne

Abstract

The solution of inverse problems involving systems modeled by partial differential equations (PDEs) is computationally demanding. We show how to take advantage of reduced-order modeling techniques to speed up the numerical approximation of Bayesian inverse problems related with parameter estimation for both stationary and time-dependent PDEs. In the former case, we rely on Markov Chain Monte Carlo (MCMC) methods to characterize the posterior distribution of the parameters; in the latter, we exploit the Ensemble Kalman filter for performing state/parameter estimation sequentially. In both cases, we replace usual high-fidelity techniques (such as the finite element method) with inexpensive but accurate reduced-order models to speed up the solution of the forward problem. On the other hand, we develop suitable reduction error models (REMs) - or ROM error surrogates - to quantify in an inexpensive way the error between the high-fidelity and the reduced-order approximation of the forward problem, in order to gauge the effect of this error on the posterior distribution of the identifiable parameters. Numerical results dealing with the estimation of both scalar parameters and parametric fields highlight the combined role played by RB accuracy and REM effectivity.

An inverse problem related to a nonlinear parabolic equation arising in electrophysiology of the heart

Luca Ratti Politecnico di Milano, Milan

Abstract

Mathematical modeling applied to the physiological description of the heart has lead to several challenging inverse problems in recent years. In particular, the inverse problem of electrophysiology consists in identifying the electrical properties of the heart tissue from non invasively measured data. The problem is non linear and severely ill-posed, but different regularization hypotheses can be introduced. In this talk, we tackle the problem of identifying the location of small regions in which the coefficients of the physiological model assume different values from the reference ones (representing early-stage ischemic regions), from the knowledge of the exact solution on the boundary. We discuss the well-posedness of the direct problem and report a one-shot reconstruction strategy for the inverse problem, which is based on the topological gradient of a suitable cost functional and exploits an asymptotic expansion of the boundary data in presence of small inclusions. We show numerical results obtained in several test cases and discuss the feasibility and the stability of the technique. This is a joint work with E. Beretta, C. Cavaterra, M.C. Cerutti and A. Manzoni

Regularisation and discretisation for the Calderón problem

Luca Rondi Dipartimento di Matematica e Geoscienze
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Abstract

We discuss the reconstruction issue for the inverse conductivity problem, in the case of discontinuous conductivities. We propose a variational approach that combines, simultaneously, regularisation and discretisation of the inverse problem. We show that the corresponding discrete regularised solutions are a good approximation of the solution to the inverse problem. The method also shows how to choose the regularisation parameter and the mesh size of the discretisation when solving numerically the inverse problem.

MS 05: Analytic and Numerical Aspects of Radon Transforms
Organizers: Todd Quinto and Peter Kuchment

Microlocal analysis of Radon transforms with cusp singularities
Raluca Felea Rochester Institute of Technology, USA

Abstract

We analyze generalized Radon transforms which exhibit a cusp singularity on the right side and a S_{1_k} singularity on the left side. We use microlocal analysis techniques to study the composition calculus of these operators and we show that their wave front set is the union of the diagonal and the open umbrella.

Non-standard limited data tomography
Jürgen Frikel OTH Regensburg

Abstract

In this talk, we report on a incomplete data problem, where the boundary of the cutoff in the sinogram domain has a triangular shape. In this case, the data truncation is dependent on the view angle as well as on the displacement variable. In particular, the curve along which the cutoff in the sinogram domain is performed is not smooth (as opposed to the cutoff in limited angle tomography). The underlying mathematical problem is significantly different from limited angle tomography and reconstructions from this kind of data show added artifacts that have different characteristics than limited angle artifacts. This particular problem is motivated by the special imaging setup that is used to examine the micro- and nanostructures of chalk samples from the North Sea in order to predict its petrophysical parameters. In this talk we will outline the differences between classical limited angle tomography and the non-standard limited data that arises from truncation with nonsmooth boundary. We will explain why and where artifacts are generated by using microlocal analysis, and present numerical experiments with real and simulated data. This is joint work with Eric Todd Quinto, Leise Borg, and Jakob Jørgensen.

Topological derivatives for domain functionals with an application to tomography
Esther Klann Johannes Kepler University Linz,
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Abstract

We study the topological sensitivity of the piecewise constant Mumford–Shah type functional for linear ill-posed problems. We consider a linear operator $K : X \to Y$ and noisy data g^δ approximating $g = Kf$ where f is the function we are interested in. We assume $f : D \to \mathbb{R}$, $D \subset \mathbb{R}^2$ and

$$f = \sum_{i=1}^{m} c_i \chi_{\Omega_i} \quad \text{with} \quad c_i \in \mathbb{R},\ \Omega_i \subset \mathbb{R}^2 \quad \text{and} \quad \chi_D = \sum_{i=1}^{m} \chi_{\Omega_i},$$

i.e., f is a piecewise constant function and the sets Ω_i are a partition of the domain of definition D. We study the topological sensitivity of the Mumford–Shah-type functional

$$J(\vec{c}, \vec{\Omega}) := \|Kf - g^\delta\|_{L_2}^2 + \alpha \sum_{i=1}^{m} |\partial \Omega_i|,$$

i.e., its reaction to a change in topology such as inserting or removing a set Ω_j. The topological derivative indicates if such a change in the topology will decrease the value of the Mumford-Shah-type functional, thus it can be used to find a minimizer of the functional and a solution to the reconstruction problem.

We use the topological derivative in an application from tomographic imaging (with the Radon transform as operator) to find inclusions in an object.

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Inversion of restricted ray transforms of symmetric rank m tensor fields in n-dimensional Euclidean space

Venky Krishnan TIFR Centre for Applicable Mathematics, Bangalore, India

Abstract

We consider the integral geometry problem of recovering rank m symmetric tensor fields from its integrals along lines in n-dimensional space. We focus on ray transforms restricted to lines passing through a fixed smooth curve. Under suitable conditions on the curve, we will present microlocal inversion results for the recovery of a component of the symmetric tensor field from its ray transform.

Singular FIOs in SAR Imaging: Transmitter and Receiver at Different Speeds

Clifford J. Nolan University of Limerick

Abstract

In this talk, we consider two particular bistatic cases which arise in Synthetic Aperture Radar (SAR) imaging: when the transmitter and receiver are moving in the same direction or in the opposite direction and with different speeds. In both cases, we classify the forward operator F_c as an FIO with singularities. Next we analyze the normal operator $F_c^* F_c$ in both cases (where F_c^* is the L^2 adjoint of F_c). When the transmitter and receiver move in the same direction, we prove that $F_c^* F_c$ belongs to a class of distributions associated to two cleanly intersecting Lagrangians, $I^{p,l}(\Lambda_1, \Lambda_2)$. When they move in opposite directions, $F_c^* F_c$ is a sum of such operators. In both cases artifacts appear and we show that they are as strong as the bona-fide part of the image. This is joint work with G. Ambartsoumian, V.P. Krishnan, R. Felea and E.T. Quinto.

Approximate inverse for the common offset acquisition geometry in 2D seismic imaging

Andreas Rieder Karlsruhe Institute of Technology, Karlsruhe, Germany

Abstract

In the inverse problem of seismology one seeks subsurface material parameters from measurements of reflected waves on a part of the propagation medium (typically an area on the earth's surface or in the ocean). To this end sources excite waves at certain positions and their reflections are recorded by receiver arrays. From a mathematical point of view we have to deal with a nonlinear parameter identification problem for a version of the elastic wave equation (with damping). This problem is solved by a multi-stage process which starts with determining the wave speed from a simpler model: the acoustic wave equation. By linearization (Born approximation) we are led to the generalized Radon transform as a model for linear seismic imaging where the sound speed is averaged over reflection isochrones connecting sources and receivers (microphones) by points of equal travel time.

In this talk we explore how the concept of approximate inverse can be used and implemented to recover singularities in the sound speed from common offset measurements in two space dimensions. Numerical experiments demonstrate the performance of the method.

This is joint work with Christine Grathwohl, Peer Kunstmann, and Todd Quinto.

MS 06: Inverse problems in optical imaging
Organizer: John C. Schotland

Inverse scattering for OCT

P. Scott Carney University of Illinois Urbana Champaign,
405 N Mathews Ave, Urbana, Illinois

Abstract

Optical coherence tomography (OCT) provides an alternative to physical sectioning that allows for imaging of living samples and even in vivo examination of cell structure and dynamics. There is, in the OCT community, a widely held belief that there exists a trade-off between transverse resolution and the thickness of the volume that may be imaged with a fixed focal plane. Efforts to overcome this trade-off have focused on the design optical elements and imaging hardware.

I present a solution of the inverse scattering problem for optical coherence tomography (OCT) called interferometric synthetic aperture microscopy (ISAM) that provides resolution everywhere equal to the best resolution in the raw data (in the focal plane). I will discuss the connection between this solution, synthetic aperture radar, and the projection-slice theorem. I will give several examples of the method in use in biological systems and results from a recent clinical trial in breast cancer.

Several import mathematical problems remain in this field. I will discuss a few of the more interesting and immediately useful examples.

On The Homogenization of a scalar Scattering problem for highly oscillating anisotropic media

Shari Moskow Drexel University, 3141 Chestnut St., Philadelphia, PA , 19104, USA

Abstract

We study the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficients modeled by the anisotropic Helmholtz equation. The coefficients are assumed to be periodic functions of the fast variable, specified over the unit cell with characteristic size ϵ. By way of multiple scales expansion, we focus on the $O(\epsilon^k)$, $k = 1, 2$, bulk and boundary corrections of the leading-order ($O(1)$) homogenized transmission problem. The analysis in particular provides the H^1 and L^2 estimates of the error committed by the first-order-corrected solution considering (i) bulk correction only and (ii) boundary and bulk correction. We treat explicitly the $O(\epsilon)$ boundary correction for the transmission problem when the scatterer is a unit square and show it has an L^2-limit as $\epsilon \to 0$, provided that the boundary cutoff of cells is fixed. We also establish the $O(\epsilon^2)$ bulk correction describing the mean wave motion inside the scatterer. The analysis also highlights a previously established, yet scarcely recognized, fact that the $O(\epsilon)$ bulk correction of the mean motion vanishes identically. This is a joint work with Fioralba Cakoni and Bojan B. Guzin.

Inverse Problems in Quantum Optics

John C. Schotland Department of Mathematics, University of Michigan, Ann Arbor, MI

Abstract

We consider the scattering of entangled two-photon states in deterministic and random media. We investigate the influence of the entanglement of the incident field on the entanglement of the scattered field. A related inverse problem is described.

MS 07: Analytic Aspects of Radon Transforms

Organizers: Todd Quinto and Peter Kuchment

Stability estimates in tensor tomography

Jan Boman Department of Mathematics, Stockholm University, 106 91 Stockholm

Abstract

For a compactly supported vector field $f = (f_1, \dots, f_n)$ in $\mathbf{R}^n$ the ray transform If is defined by

$$If(x,\xi) = \sum_{j=1}^{n} \int_{\mathbf{R}} f_j(x+t\xi)\xi_j dt, \quad (x,\xi) \in \mathbf{R}^{2n},$$

and for a symmetric 2-tensor field $(f_{jk})_{j,k=1}^n$ one defines

$$If(x,\xi) = \sum_{j,k=1}^{n} \int_{\mathbf{R}} f_{jk}(x+t\xi)\xi_j\xi_k dt, \quad (x,\xi) \in \mathbf{R}^{2n}.$$

Since $If = 0$ if f is the gradient of a scalar function, one can at most recover the so-called solenoidal part ${}^sf$ of f from If. An analogous fact is true for second and higher order tensor fields. Similarly, if f is only defined in a bounded convex subset $\Omega \in \mathbf{R}^n$, there is a natural definition of the solenoidal part ${}^sf$ of f relative to Ω. For tensor fields of order 1 and 2 in Ω we give estimates for the norm of ${}^sf$ of the type

$$\|{}^sf\| \leq C\|If\|_{1/2},$$

where $\|\cdot\|$ is the L^2-norm and $\|\cdot\|_{1/2}$ is a Sobolev norm of order 1/2. Similar, but somewhat weaker, estimates were given in Vladimir Sharafutdinov's book *Integral geometry of tensor fields* (Utrecht 1994), for the much more general case of a ray transform with respect to a Riemannian metric and for tensor fields of arbitrary order under a certain curvature condition on the boundary of Ω. This is joint work with Vladimir Sharafutdinov.

Curving of Radon's inversion formula

Simon Gindikin Departm. of Math., Hill Center, Rutgers University,
110 Frelinghysen Road, Piscataway, NJ 08854-8019,U.S.A.

Abstract

Abstract. On the space of (local) parameterized curves on the plane we build such a variational operator on functionals that if the functional is the integral of a function over the curves then the image is a closed form and its integrals over cycles give the value of this function at the origin up to a constant depending of the cycle. By choices of cycles we obtain different inversion formulas including the classical Radon formula and its hyperbolic versions. A generalization of this construction gives the inversion of the horospherical transform on symmetric spaces.

Mean Value Operators on Noncompact Symmetric Spaces

Fulton Gonzalez Tufts University, USA

Abstract

Fix $r > 0$. The mean value operator

$$M^r f(x) = \frac{1}{A(r)} \int_{S_r(x)} f(y)\, dm(y) \qquad (f \in C(\mathbb{R}^n))$$

can be viewed as a Radon transform on functions on $\mathbb{R}^n$, with respect to the incidence relation under which two points in $\mathbb{R}^n$ are incident iff they are at distance r apart. It can also be viewed as a convolution operator $M^r f = f * \mu_r$, where μ_r is the distribution integrating test functions on the sphere of radius r centered at the origin. We consider the mapping properties of this transform, as well as the more general mean value operator

$$M^y f(gK) = \int_K f(gk \cdot y)\, dk \qquad y \in X,\, g \in G,\, f \in C(X)$$

on a symmetric space $X = G/K$. We show that this mean value operator is surjective in many specific cases. The key tool is Ehrenpreis' slow decrease condition for compactly supported distributions as well as the projection-slice theorem for the horocycle Radon transform on X. (This is joint work with Jens Christensen and Tomoyuki Kakehi.)

Microlocal analysis of photoacoustic tomography revisited

Linh Nguyen University of Idaho, Moscow, ID 83844, USA

Abstract

Photoacoustic tomography is an emerging method of imaging. Its mathematical model is an inverse problem of the wave equation. Namely, we need to invert the operator Λ that sends the initial condition to the trace of the solution on a surface. In this talk, we analyze the adjoint of Λ from microlocal analysis point of view. We will compare and contrast it with the well-known time reversal operator. We finally describe the implication of our analysis for the inversion formulas of the spherical Radon transform.

The geodesic X-ray transform with conjugate points

Plamen Stefanov Department of Mathematics, Purdue University

Abstract

We present recent results with G. Uhlmann and F. Monard on the geodesic X-ray transform on Riemannian surfaces with conjugate points. We show that artifacts are unavoidable and describe them microlocally. We show that a positive attenuation stabilizes the problem and allows reconstructions with no artifacts if there are no more than two conjugate points along each geodesics but not of there are three or more. We will present numerical simulations to illustrate the results.

In particular, our results imply that the linearized traveltime problem of recovery a sound speed is always unstable in two dimensions if there are conjugate points. It is known that it is stable otherwise.

MS 08: Linear and non-linear tomography in non Euclidean geometries

Organizers: Plamen Stefanov, Francois Monard, and Gunther Uhlmann

Exploring generic conditions for stability of the geodesic ray transform

Sean Holman University of Manchester, School of Mathematics,
Oxford Road, Manchester M13 9PL, United Kingdom

Abstract

It has now been known for a few years that there cannot be Lipschitz type stability estimates between any Sobolev norms for inversion of the geodesic ray transform on Riemannian surfaces containing conjugate points. Since conjugate points occur generically this means that in two dimensions the ray transform is not generically stable. In this talk we will look at conditions under which there are Lipschitz type stability estimates between Sobolov spaces in three and higher dimensions, and whether these conditions occur generically.

On Marked boundary rigidity for surfaces

Marco Mazzucchelli École Normale Supérieure de Lyon,
46 alle d'Italie, 69364 Lyon, France

Abstract

In this talk, which is based on a joint work with Colin Guillarmou, I will present two boundary rigidity results for Riemannian surfaces possibly with non-trivial topology and non-empty trapped set for their geodesic flows. The first result asserts that two sufficiently C^2-close Riemannian metrics on an oriented compact surface with strictly convex boundary, no conjugate points, hyperbolic trapped set for their geodesic flows, and same marked boundary distance, are isometric via a diffeomorphism that fixes the boundary. The second result, which extends theorems of Croke and Otal, asserts the same conclusion for any two negatively curved Riemannian metrics on a compact surface with strictly convex boundary and same marked boundary distance.

The light ray transform and applications

Lauri Oksanen University College London

Abstract

The light ray transform of a function on a Lorentzian manifold is the map giving the integrals of the function over each light ray (that is, null geodesic). We show that the light ray transform is invertible when the Lorentzian manifold is the product of the real line and a Riemannian manifold on which the geodesic ray transform is invertible. Moreover, we discuss the microlocal structure of the normal operator associated to the light ray transform. We give applications to hyperbolic inverse boundary value problems and to the study of Cosmic Microwave Background measurements. The talk is based on joint works with Yavar Kian and with Matti Lassas, Plamen Stefanov and Gunther Uhlmann.

Effective inversion of the attenuated X-ray transform associated with a connection.

Gabriel Paternain Department of Pure Mathematics and Mathematical Statistics, University of Cambridge, Cambridge CB3 0WB, United Kingdom

Abstract

I will discuss inversion formulas (up to a compact error) for the attenuated X-ray transform on a non-trapping surface, when the attenuation is determined by a complex matrix of 1-forms (the connection). The formulas lead naturally to a filtered-backprojection algorithm. This is joint work with François Monard.

Generic uniqueness and stability of inverse problems for connections

Hanming Zhou Department of Pure Mathematics and Mathematical Statistics, University of Cambridge, Cambridge CB3 0WB, United Kingdom

Abstract

We consider the nonlinear problem of determining a connection and a Higgs field from the corresponding parallel transport along geodesics on a compact Riemannian manifold with boundary, in any dimension. The problem can be reduced to an integral geometry question of some attenuated geodesic ray transform through a pseudolinearization argument. We show injectivity (up to natural obstructions) and stability estimates for both the linear and nonlinear problems for generic simple metrics and generic connections and Higgs fields, including the real-analytic ones. We consider the problems on simple manifolds in order to make the exposition of the main ideas clear and concise, many results in this talk still hold under much weaker geometric assumptions, in particular conjugate points and trapped geodesics are allowed and the boundary is not necessarily convex.

MS 09: Cone/Compton transforms and their applications

Organizers: Gaik Ambartsoumian and Fatma Terzioglu

Inversion of the attenuated conical Radon transform

Rim Gouia-Zarrad American University of Sharjah
PO Box 26666, Sharjah, UAE

Abstract

Since Compton cameras were introduced in the use of single photon emission computed tomography (SPECT), various types of conical Radon transforms, which integrate the emission distribution over circular cones, have been studied. Most of previous works did not address the attenuation factor, which may lead to significant degradation of image quality. In this presentation, we consider the problem of recovering an unknown function from conical projections affected by a constant known attenuation coefficient called attenuated conical Radon transform. In the case of fixed opening angle and vertical central axis, new explicit inversion formulae are derived. In addition, the results of numerical simulations are presented for the 2D case.

Compton camera imaging and Radon transforms on the cone: from data to images

Voichita Maxim Universit de Lyon, CREATIS, CNRS UMR5220,
Inserm U1044, INSA-Lyon, Universit Lyon 1, France

Abstract

The recent advances in Compton camera imaging led to new integral transforms that may be assimilated to Radon transforms on cones. We mainly have two classes of models. The first one consists in integrating the intensity function on the cone's surface and in the second one the activity measured on the set of generatrices is summed. The inverse transform either determines directly the unknown intensity function or its integrals on half-lines. For the first approach some filtered back-projection algorithms were proposed. The second approach is well adapted for far-field imaging and for mono-pixel detectors.

The diversity of image reconstruction methods in Compton imaging is facilitated by the over-dimensioned data space. As an example, with a single device composed of two infinite extent planar detectors, the data space has five dimensions and the image space only three. The redundancy in the data space can be used to reduce the noise in the image. It is important to exploit all the available data because in practical applications the goal is to produce an image of the source with as few photons as possible, and the available photons are dispersed in a five dimensional space.

Another important issue is the completeness of the data space. The detectors have finite extent and a practical geometry leading to a complete data set has not been found yet.

Which model better suits the data in Compton imaging? Which acquisition geometry is less subject to reconstruction artefacts produced by presumably unavoidable projection truncations? How could redundancy be better exploited? Of course, the answer is not simple, and depends on the way the data is acquired and on the foreseen application.

In this work the emphasis will be put on planar acquisition geometries. First, a brief discussion of the models usually employed in Compton camera imaging will be carried, with the support of theoretical arguments and Monte-Carlo simulation results. Then a filtered back-projection algorithm will be presented. Finally, the influence of the model on the reconstructed images will be shown.

Numerical Inversion of a Broken Ray Transform Arising in Single Scattering Optical Tomography

Dr. Souvik Roy Postdoctoral fellow,
Group of Scientific Computing,
University of Würzburg, Würzburg-97074, Germany

Abstract

In this talk, we present an efficient image reconstruction algorithm for single scattering optical tomography (SSOT) in circular geometry of data acquisition. This novel medical imaging modality uses photons of light that scatter once in the body to recover its interior features. The mathematical model of SSOT is based on the broken ray (or V-line Radon) transform (BRT), which puts into correspondence to an image function its integrals along V-shaped piecewise linear trajectories. The process of image reconstruction in SSOT requires inversion of that transform. We implement numerical inversion of a broken ray transform in a disc with partial radial data. Our method is based on a relation between the Fourier coefficients of the image function and those of its BRT recently discovered by Ambartsoumian and Moon. The numerical algorithm requires solution of ill-conditioned matrix problems, which is accomplished using a half-rank truncated singular value decomposition method. Several numerical computations validating the inversion formula are presented, which demonstrate the accuracy, speed, and robustness of our method in the case of both noise-free and noisy data.

This is a joint work with Gaik Ambartsoumian.

The Radon transform over cones with vertices on the sphere and orthogonal axes

Daniela Schiefeneder Department of Mathematics, University of Innsbruck

Abstract

Recovering a function from its integrals over circular cones recently gained significance because of its relevance to novel medical imaging technologies such as emission tomography using Compton cameras. In this talk we investigate the case where the vertices of the cones of integration are restricted to a sphere in n-dimensional space and symmetry axes are orthogonal to the sphere. We show invertibility of the considered transform and develop an inversion method based on series expansion. For that purpose we derive a novel uniqueness result for generalized Abel equations where the kernel has zeros on the diagonal. Finally, we present numerical results using our inversion method.

This is joint work with Markus Haltmeier.

MS 10: Vector and tensor tomography: advances in theory and applications

Organizer: Thomas Schuster

Radon transform of vector fields and applications to partial differential equations

Kazantsev Sergey Gavrilovich

Sobolev Institute of Mathematics, 4 Acad.
Koptyug prosp., 630090 Novosibirsk, Russia

Abstract

In this talk, we shall consider the relation of the componentwise Radon transform of a vector field in a 3D bounded domain with the classical integral theorems of vector analysis. The integral identities containing the Radon transform and differential operators of vector fields (divergence, curl, gradient) are obtained. The image of the componentwise Radon transform is also a vector field, but now one is defined on the unit sphere. Therefore, it is convenient to calculate the normal and tangential components of the image vector field. It is known, that Helmholtz decomposition split any smooth vector field into a divergent-free part, curl-free, and a harmonic part. The last part is both divergence-free and curl-free. The main problem is to determine the harmonic part, see [1],[2]. We propose here a new algorithm for Helmholtz decomposition in which we use only the orthogonal projection on the unit sphere, Radon and inverse Radon transforms (three times).

We also shall discuss the solving of boundary value problems for Maxwell's equations (div-curl system) via Radon transform. The results can also be applied to problems of vector tomography on an arbitrary bounded domain in three dimensions.

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2. N.F. Osman and J. L. Prince, 3D vector tomography on bounded domains. Inverse Problems, **14**(1998), no 1, 185–196.

Towards the reconstruction of focal electrical brain activity with the help of vector tomography

Alexandra Koulouri Institute for Computational and Applied Mathematics,
University of Münster, Einsteinstrasse 62, D-48149 Münster, Germany

Abstract

In vector tomography (VT) unknown multi-dimensional vector fields are reconstructed by using line integral measurements. In this work, we describe how vector tomography framework can be used to reconstruct 2-dimensional electric fields induced by sparse focal sources in a convex domain with homogeneous Neumann boundary condition. This case often occurs in electroencephalography focal source imaging. In contrast to divergence-free (or source-free) vector fields which are much easier to solve, we study electric fields with non-zero divergence. Now two types of Radon measurements, i.e. the longitudinal and transverse integral data, are required for a full recovery. Unfortunately, in this problem the transverse measurements cannot be physically acquired. In this study, we show that these non-zero divergence electric fields can be reconstructed from the longitudinal measurements with the help of two sparsity constraints that are constructed based on the transverse integrals and the vector Laplace operator. We formulate this as a convex minimization problem and finally, we use simulated test cases to validate our approach. This is a joint work with Mike Brookes and Ville Rimpiläinen.

Tensor tomography on Riemannian manifolds

Mikko Salo University of Jyväskylä

Abstract

The Euclidean X-ray transform, which encodes the integrals of a function over straight lines, is a classical topic going back to J. Radon in 1917 and forms the basis of imaging methods such as X-ray computed tomography. The geodesic X-ray transform encodes the integrals of a function or a tensor field over more general families of curves, such as the geodesics of a Riemannian metric. It has applications in seismic imaging and in geometric inverse problems including the boundary rigidity problem, the inverse conductivity problem posed by Calderón, and inverse spectral problems.

We will discuss recent theoretical advances in vector and tensor tomography on compact Riemannian manifolds with boundary, based on joint works with C. Guillarmou, G. Paternain, G. Uhlmann and H. Zhou.

On the application of a Hilbert transform to tensor tomography

Alexandru Tamasan University of Central Florida,
Orlando, Florida, USA

Abstract

In this talk I will recall Bukhgeim's approach to the tomography problem in the plane and exemplify it by the reconstruction and range characterization of the attenuated X-ray transform of symmetric tensors. This is joint work with Kamran Sadiq and Otmar Scherzer.

MS 11: Applications of the Radon Transform
Organizer: Simon Arridge

Joint image reconstruction of absorption and refraction properties in single-shot edge-illumination X-ray phase-contrast tomography

Mark A. Anastasio Washington University in St. Louis,
Department of Biomedical Engineering, St. Louis, MO, U.S.A.

Abstract

Edge illumination X-ray phase-contrast tomography (EIXPCT) is an emerging imaging technique that seeks to circumvent the limitations of previous benchtop implementations of X-ray phase-contrast tomography. The goal of EIXPCT is to produce images that separately depict the spatially variant X-ray refractive index and absorption distributions within an object. As with grating- or analyzer-based methods, conventional image reconstruction methods for EIXPCT require that two or more images are acquired at each tomographic view angle. This requirement leads to increased data-acquisition times and radiation doses, which can hinder in vivo applications. To circumvent this, a joint reconstruction (JR) approach is proposed that concurrently produces estimates of the refractive index and absorption distributions from a tomographic data set containing only a single image per tomographic view angle. The JR reconstruction method solves a non-linear optimization problem by use of a novel iterative gradient-based algorithm. The JR method is validated and investigated by use of both computer-simulated and experimental EIXPCT data. This work involved a collaboration between the author and Yujia Chen, Huifeng Guan, Charlotte K. Hagen, and Alessandro Olivo.

The Radon transform for dynamic objects

Bernadette Hahn University of Würzburg

Abstract

The Radon transform provides the mathematical basis for many imaging modalities, such as CT, MRI, etc. In many applications, the investigated object might change during the data acquisition process, e.g. in medical imaging due to patient motion, or in non-destructive testing while imaging driven liquid fronts. Such temporal changes lead to inconsistent Radon data and, hence, the application of standard reconstruction techniques based on the inversion of the Radon transform causes motion artefacts in the images which can severely impede a reliable diagnostics.

Incorporating the time-dependency of the specimen alters the underlying mathematical model, leading to an inverse problem involving integral transforms over curved lines. In this talk, we study some properties of these Radon-type transforms and discuss their consequences for practical applications.

Region-of-Interest CT with Limited Data for Underwater Pipeline Inspection

Per Christian Hansen DTU Compute, the Department of Applied Mathematics and Computer Science, Technical University of Denmark

Abstract

This project concerns the inspection of defects in underwater pipelines using X-ray computed tomography (CT). We study which mathematical models and reconstruction techniques are best suited for limited-angle CT, in which only the central region of the pipe - the region-of-interest (ROI) - is fully illuminated. Of particular interest are defects, such as rust and cracks, near the edge of the ROI.

The goal is to gain insight into the inherit difficulties and limitations of this problem, study the resolution limits and the uncertainties of the reconstruction and how these factors depend on the geometry, and determine which state-of-the-art models and methods are best suited for the problem.

This is joint work with Assoc. Prof. Yiqiu Dong and PhD candidate Rasmus Dalgas Kongskov, as well as MSc students Jacob Frsig and Nicolai Andre Brogaard Riis, from DTU Compute.

The project is a collaboration with FORCE Technology, Denmark.

Non-linearity in monochromatic transmission tomography

William Lionheart School of Mathematics, University of Manchester, UK

Abstract

While it is well known that X-ray tomography using a polychromatic source is non-linear, as the linear attenuation coefficient depends on the wavelength of the X-rays, tomography using near monochromatic sources are usually assumed to be a linear inverse problem. When sources and detectors are not treated as points the measurements are the integrals of the exponentials of line integrals and hence non-linear. In this talk we show that this non-linearity can be observed in realistic situations using both experimental measurements in a γ-ray tomography system and Monte Carlo Simulations.

We exhibit the Jacobian matrix of the non-linear forward problem and show that the non linear inverse problem can be iteratively. Applying this algorithm to experimental data we show that improved reconstructions can be obtained. We discuss the mathematical problem of uniqueness of solution.

This is joint work with Sophia B. Coban, Bjørn Tore Hjertaker Rachid Maad, Ilker Meric, and Geir Anton Johansen

Algorithm-Enabled Imaging in CT and PET

Xiaochuan Pan The University of Chicago, 5841 S. Maryland Avenue, MC2026 Chicago, IL 60637, USA

Abstract

In conventional development of tomographic imaging such as CT and PET, the hardware and algorithm components are designed and developed based often upon a sequential approach. There is an increasing level of recognition that the design of hardware by incorporation of algorithm consideration can have implication for the development of low-cost hardware/algorithm system of practical utility tailored most appropriately to addressing applications of significance. The focus of work is on analysis and demonstration of the possibility for developing reconstruction algorithms that can enable hardware design with low-cost, high adaptability, and increased utility for applications. Specifically, optimization-based reconstruction, which encompasses optimization problem and algorithm solving the problem, and its application to enabling system design and/or practical workflow driven by applications will be discussed. Examples will be used for illustrating how optimization algorithms can be leveraged for realizing advanced CT and PET systems tailored to specific design constraints and application purposes.

Sparse-data tomographic imaging in practice

Samuli Siltanen Department of Mathematics and Statistics,
University of Helsinki, Finland

Abstract

Tomographic reconstruction from comprehensive projection data is a mildly ill-posed inverse problem which is already well-understood. By comprehensive projection data we mean a large number of radiographs taken from essentially all directions around the target. However, many practical imaging situations allow only sparsely collected data. Reasons for this may include desire to reduce radiation dose or data collection time, or mechanical restrictions on imaging directions. Tomographic reconstruction from sparsely collected data is a severely ill-posed problem, and it needs to be regularized by complementing the measurement information by *a priori* knowledge about the target. The shearlet transform provides a flexible and computational efficient way of enforcing piecewise smoothness in the attenuation coefficient. This is relevant in many practical applications of tomography. Shearlet-sparsity-promoting regularization is discussed and illustrated with both static and dynamic tomography examples. In particular, the examination of bone samples for detecting osteoporosis can be speeded up by a factor of 20 using shearlet-based methods.

MS 12: Numerical microlocal analysis
Organizers: Marta Betcke and Jürgen Frikel

A Support Theorem for Integral Moments of a Symmetric m-Tensor Field

Anuj Abhishek Tufts University, Medford, MA - 02155, USA

Abstract

In this talk I will present a brief account of a recent work done jointly with Rohit Kumar Mishra. I will first define what we mean by the q-th integral moment, $I^q f$, of any symmetric tensor field f of order m over a simple, real-analytic Riemannian manifold of dimension n. The zeroth integral moment of such tensor fields coincides with the usual notion of geodesic ray transform of the tensor fields. Then I will introduce some tools from geometry and microlocal analysis that we use to prove our result. Finally I will give an outline of the proof for our main theorem which tells us that if we know $I^q f = 0$ for $q = 0, 1, \ldots, m$ over the open set of geodesics not intersecting a geodesically convex set, then the support of f lies within that convex set.

Structured guided Total Variation

Marta M. Betcke University College London, United Kingdom

Abstract

In this talk we discuss two modifications of total variation based on (i) location and (ii) direction that take structural a priori knowledge into account and reduce to total variation in case when no structural information is available. We solve the resulting convex minimization problem with the alternating direction method of multipliers which separates the forward operator from the prior. For both priors the corresponding proximal operator can be implemented as an extension of the fast gradient projection method on the dual problem for total variation. We demonstrate the effectiveness of such a direction informed total variation on the multi-contrast MRI data. This is a joint work with Matthias Ehrhardt.

Wavelet methods in photoacoustic tomography

Markus Haltmeier Department of Mathematics, University of Innsbruck, Technikerstrasse 13, A-6020 Innsbruck, Austria

Abstract

Photoacoustic tomography (PAT) is a recently developed coupled physics imaging modality that combines the high spatial resolution of ultrasound imaging with the high contrast of optical imaging. Recovering a diagnostic image in PAT requires the solution of an inverse problem for the acoustic wave equation. In this talk we develop wavelet methods for that purpose. We investigate the case of deterministic as well as random noise, establish a numerical algorithm, and derive optimal error estimates for our proposal. This is joint work Jürgen Frikel.

Microlocal stability and instability in Acousto-Electric tomography

Kim Knudsen Technical University of Denmark, Department of Applied Mathematics and Computer Science, Matematiktorvet Building 303B, 106, DK-2800 Kgs. Lyngby

Abstract

The reconstruction problem in Acousto-Electric tomography (AET) aims at recovering the interior conductivity distribution in an object from electrostatic data measured on the object's boundary. During the measurement process the object is probed by ultrasonic waves which perturb the conductivity slightly. By controlling the wave propagation the electrostatic measurements allow for the computation of the interior power functional which in turn under favorable conditions allow high contrast and high resolution reconstructions of the conductivity.

In this talk the mathematical problem of Acousto-Electric Tomography (and similar kinds) will be introduced. The fundamental questions will be posed and (partially) answered. In particular we will discuss instability issues related to the propagation of singularities, suggest remedies for stabilization, and illustrate the concepts through numerical examples.

Total Variation Regularization in variable Lebesgue spaces

Holger Kohr Centrum Wiskunde & Informatica (CWI),
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Abstract

Variable Lebesgue spaces, i.e., L^p spaces with spatially varying exponent p, along with corresponding Sobolev spaces, are by now fairly well understood from a Harmonic Analysis point of view. Recently, also the variable-exponent variants of spaces of Bounded Variation functions (BV spaces) have been studied in the context of PDEs [1, 3] and image restoration [2]. However, regularization in practice using variable exponent total variation priors has experienced limited use to the date [7, 4], and so far only results with smoothed versions of the TV functional have been given.

The main motivation for using a variable exponent in TV regularization is the freedom to use additional domain knowledge to steer the penalization of image features depending on the image region. In a tomographic setting, the following workflow would be possible: First, an edge image is reconstructed, using e.g. Lambda tomography [6] or Approximate Inverse with a derivative-type feature operator [5]. From this edge image, the exponent function is extracted as a function taking values between 1 and 2, where 1 is preferred for image parts that are edge-like, and 2 for others. Finally, this exponent is then used in variable exponent TV regularization to acquire the final reconstruction.

This talk will introduce the theoretical concepts and demonstrate properties of this regularization scheme with numerical examples.

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Artifacts in Arbitrary Limited Data Tomography Problems

Eric Todd Quinto Tufts University, Medford, MA, USA

Abstract

In this talk, we will describe work of the speaker along with Leise Borg, Jürgen Frikel, and Jakob Jørgensen characterizing how artifacts appear in limited data tomography with arbitrary data sets. We characterize artifacts in the reconstruction that are created by lines in the boundary of the data set, both when the boundary is smooth and when it is not. We apply this to standard and non-standard limited data tomography problems with real and simulated data. We outline the proof, which is based on microlocal analysis. This work is motivated by an unusual synchrotron CT data set that caused streaks in the reconstruction that were not explained by the previous theory which will be discussed by Jürgen Frikel.

MS 13: Radon-type transforms: Basis for Emerging Imaging

Organizers: Bernadette Hahn and Gaël Rigaud

A generalization of the Funk-Radon transform

Michael Quellmalz Technische Universität Chemnitz

Abstract

The Funk-Radon transform, also known as the spherical Radon transform, assigns to a function on the sphere its mean values along all great circles. Since its invention by Paul Funk in 1911, the Funk-Radon transform has been generalized to other families of circles as well as to higher dimensions. In case of the d-dimensional sphere ($d > 3$), the great circles are replaced by subspheres being the intersection of the sphere with hyperplanes containing the origin.

We are particularly interested in the following generalization: we consider the intersections of the sphere with (hyper-)planes containing a common point inside the sphere. We give an injectivity result for this generalized Radon transform by relating it to the classical Funk-Radon transform.

Feature reconstruction in Compton scattering tomography

Gaël Rigaud Department of Mathematics, Saarland University,
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Abstract

Compton scattering tomography (CST) is an arising imaging concept exploiting the scattering radiation as a specimen of interest is illuminated by a gamma source. In the last decade the study of first-order scattered photons has led to model the measured flux arriving according the energy by Radon transforms over circles. Such transforms were shown to be invertible due to their strong relation with the classical Radon transform, i.e. the line integral transform. However, the model neglects physical phenomena : attenuation factor, photometric dispersion and strong Poisson noise. A microlocal analysis of the considered operators proves that the singularities (for example the contours) are preserved when the physical factors are taken into account. Thence, a reconstruction method designed for finding the features of the sought-for object, for instance the discontinuities, stands for a suited first step to solve image reconstruction in Compton scattering tomography. Due to the high order of ill-posedness induced by the singularities, we derive a regularized feature reconstruction scheme based on the Approximate Inverse invented by Louis.

Kaiser-Bessel functions in photoacoustic tomography

Johannes Schwab Department of Mathematics, University of Innsbruck,
Technikerstraße 13, 6020 Innsbruck, Austria

Abstract

Photoacoustic tomography (PAT) is a recently developed hybride imaging technique that is based on the photoacoustic effect. When a semitransparent sample is illuminated with a short pulse of electromagnetic energy, parts of the energy will be absorbed inside the sample which causes a rapid increase of temperature. This increase of temperature yields a spatially varying thermoelastic expansion, which in turn induces an acoustic pressure wave. The acoustic pressure wave is measured outside of the object and the mathematical task is to recover an image of the interior. In this talk we derive efficient Galerkin methods for image reconstruction in PAT, using translation invariant reconstruction spaces generated by generalized Kaiser-Bessel functions. This is joint work with Markus Haltmeier and Sergiy Pereverzyev Jr.

Semi-discrete Landweber-Kaczmarz method for limited data Cone Beam tomography

Jonas Vogelgesang Institute of Applied Mathematics, Saarland University, Campus E1 1, 66123 Saarbrücken, Germany

Abstract

In practice, many imaging modalities used in non-destructive testing (NDT) like computed tomography (CT) or computed laminography (CL) suffer from incomplete data. In particular, while inspecting objects with extremely different diameters in longitudinal and transversal directions, the architecture of the scanning system itself as well as the use of non-standard curves in CL applications lead to restricted source problems and truncated projections, also known as region-of-interest (ROI) problem. To this end, the semi-discrete Landweber-Kaczmarz method for solving linear ill-posed problems in Hilbert spaces is proposed using a basis function-type discretization in the operator domain. Exploiting this discretization, an approximation of the searched-for solution is computed iteratively in suitable subspaces generated by the basis functions. The proposed method inherently allows the incorporation of prior information, like material properties or geometrical information obtained from a second imaging modality, to compensate the lack of information in the measured data and to overcome the ROI problem. Moreover, the physical conditions of the scanning geometry, like the use of non-regular scanning curves as well as flat detectors in NDT, are directly incorporated into the reconstruction process. Finally, numerical simulations for three-dimensional applications with inherent restricted data and ROI problems are provided using material properties and contour information of the object to verify the proposed method.

Radon Transform via Machine Learning

Ge Wang Director of Biomedical Imaging, BME & CBIS, Rensselaer Polytechnic Institute Troy, NY, USA

Abstract

The Radon transform is widely used for tomographic imaging. Due to the penetrating power, fine resolution, complementary contrast mechanisms, high-speed, and cost-effectiveness of the x-ray technology, computed tomography (CT) is one of the earliest and most popular imaging modalities in biomedical and other fields. The recent advancement in deep learning, or machine learning in general, promises to perform and invert the Radon transform in innovative fashions. This direction might lead to intelligent utilization of domain knowledge from big data, innovative approaches for image reconstruction, and superior performance in important applications. In addition to a general perspective (Ge Wang, A Perspective on Deep Imaging, IEEE Access 4: 8914 8924, 2016; http://ieeexplore.ieee.org/document/7733110), some deep imaging results at our Biomedical Imaging Center will be also discussed.

MS 14: Theory and numerical methods for inverse problems and tomography

Organizer: Michael V. Klibanov

Kirchhoff migration without phases

Patrick Bardsley University of Texas at Austin, USA

Abstract

Scatterers in a homogeneous medium can be imaged using the Kirchhoff migration functional. This imaging method generally requires full waveform measurements of the scattered field and thus intensity (i.e., phaseless) measurements are insufficient to image with. However, if the scattered field is small compared to the probing field, we can solve a simple least-squares problem to recover the projection (on a known subspace) of the full waveform scattered field from intensity data. For high frequencies, this projection gives a Kirchhoff image asymptotically identical to the Kirchoff image obtained from full waveform data. This imaging method also works when the illuminating wavefields are stochastic and we measure autocorrelations at receivers.

A phaseless inverse scattering problem for the 3-D Helmholtz equation

M. V. Klibanov Department of Mathematics and Statistics,
University of North Carolina at Charlotte,
Charlotte, NC 28213, USA

Abstract

We will present our recent reconstructions procedures for 3-D inverse scattering problems without the phase information. Some procedures end up with the Radon transform. Computational results will be presented as well.

Spherical means and Godunov regularization in thermoacoustics

Olga Krivorotko Institute of Computational Mathematics and
Mathematical Geophysics, Novosibirsk, Russia

Abstract

We will present our recent reconstructions of inhomogeneity for 2D inverse thermoacoustic problems that is important in cancer diagnostic. We show the condition of stability of inverse problem using spherical means. Also we apply optimization techniques and Godunov regularization for computing the inverse problem solution. Computational results will be presented as well.

Numerical solution for an inverse coefficient problem with single measurement of multi-frequency data.

Loc Nguyen University of North Carolina Charlotte, USA

Abstract

The goal of this talk is to reconstruct spatially distributed dielectric constants from complex-valued scattered wave fields by solving a 3D inverse medium (or coefficient) problem for the Helmholtz equation at multi-frequencies. The data are generated by only one direction of a incident planewave. To solve this inverse problem, a globally convergent algorithm is analytically developed. We prove that this algorithm provides a good approximation for the true solution without any *a priori* knowledge of any point in a small neighborhood of the exact coefficient. This is the main advantage of our method, compared with classical approaches using optimization schemes. Numerical results are presented. The main focus of numerical tests is on the case when only the backscatter data are available. In addition to computationally simulated data, results for experimental data are presented as well. The potential applications of this problem are in detection and identification of explosive-like targets.

The importance of the Radon transform in vector field tomography

Thomas Schuster Department of Mathematics,
Saarland University, Saarbrücken, Germany

Abstract

Vector field tomography means to recover a vector field, such as a velocity field of a moving fluid, from ultrasound measurements. The mathematical model of this inverse problem is the longitudinal ray transform. These are line integrals of the field projected along the direction of the line. It is quite obvious that in 2D this transform is directly related to the 2D Radon transform. But it is surprising that the Radon transform plays also a prominent role for inversion methods in the cone beam geometry for the 3D vector field tomography problem. In the talk we present the tight connection between the Radon transform and vector field tomography, show a new inversion formula for the cone beam geometry and numerical results.

The research presented in the talk is joint work with Alexander Katsevich (University of Central Florida, Orlando, USA).

Spectra of Running Waves and Solution to Inverse Problems in Waveguides

Yury Shestopalov Faculty of Engineering and Sustainable Development,
University of Gävle, 801 76 Gvle, Sweden

Abstract

The running waves and eigenoscillations in waveguides are studied using the reduction to spectral problems that describe singularities of the analytical continuation of the solutions to the problems with sources in the complex domain, a multi-sheet Riemann surface where the spectral parameter is varied.

We consider (i) the forward problem of the electromagnetic wave scattering by a layered parallel-plane dielectric diaphragms in a waveguide of rectangular cross section, and (ii) the inverse problem of reconstructing permittivity of such multi-layer dielectric inclusions. It is shown that permittivity can be uniquely determined from the transmission coefficient (measured at several frequencies) considered as a one-to-one function of the appropriate complex variable(s).

Performing the analysis of the forward scattering problem in the waveguide, we show [1] that the transmission coefficient extended to the complex domain of some of the problem parameters (permittivity of one of the layers of the inclusion, longitudinal wavenumber, or frequency) has singularities–finite-multiplicity poles–associated with eigenvalues of distinct families of Sturm–Liouville problems on the line; they form countable sets of points in the complex plane with the only accumulation point at infinity and depend continuously on the problem parameters. Classification and interpretation of the corresponding solutions to the boundary value problems for the Maxwell equations are given: the corresponding eigensolutions have the form of standing waves. The singularities can be calculated with prescribed accuracy by explicit formulas using the (numerical) solution to simple transcendental equations. The set of complex eigenfrequencies is similar in its structure to the set of eigenvalues of the Laplacian in a rectangle.

The discovered singularities of the transmission coefficient are called resonant states (RSs), by analogy with the quantum scattering theory. The knowledge of RSs enhances the possibilities of solving both the forward problems of the waveguide filter design and inverse problems of reconstructing permittivity of multi-layer parallel-plane diaphragms and more complicated dielectric scatterers in the waveguide.

To solve the inverse problems of reconstructing permittivities and permeabilities of the layered parallel-plane inclusions, a numericalanalytical technique is developed [2] employing an inversion of the operator of the forward problem and information about its spectrum.

1. Y. V. Shestopalov, Resonant states in waveguide transmission problems, *Progress In Electromagnetics Research B, Vol. 64, pp. 119-143 (2015).*

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MS 15: Towards Robust Tomography
Organizer: Samuli Siltanen

Iterative ℓ_1 shearlet regularization for the ROI tomography problem

Tatiana A. Bubba Department of Mathematics and Statistics, University of Helsinki, Finland

Abstract

In the last years, region-of-interest tomography (ROI CT) established itself among the limited data strategies, in an era of increasing attention to biomedical applications that allow to lower exposure to X-ray radiation, as well as shortening the scanning time. However, the truncations of the projection measurements, at the basis of ROI CT, leads to a severe ill-posedness. Since traditional CT reconstruction algorithms result in instability to noise, and might give inaccurate results, especially for small ROIs, *ad hoc* strategies are necessary for accurate, reliable and robust reconstructions. To this end, we propose a regularized solution, expressed by means of a nonsmooth convex optimization model based on ℓ_1 shearlet regularization. Namely, we minimize a functional of the form

$$\min_{\mathbf{f}\in\mathbb{R}^n} \Gamma_0(\mathbf{f}) + \Gamma_1(\mathbf{f}) \tag{2}$$

where $\mathbf{f}$ is the object to be reconstructed, Γ_0 is the data mismatch term, and Γ_1 includes a priori information on the solution, by imposing sparsity (in the sense of the ℓ_1-norm) on the shearlets coefficients of the missing data. The solution of (2) is addressed by means of the very recently proposed variable metric inexact line-search algorithm (VMILA), a proximal-gradient method that enables the inexact computation of the proximal point defining the descent direction. We compare the reconstruction performance of our strategy against a smooth total variation (sTV) approach, by using both noisy simulated data and real data. The goal is to show that, while for synthetic data both shearets and sTV perform similarly, for real data, the proposed nonsmooth shearlet-based approach outperforms sTV, yielding more accurate reconstructions, which are also more robust to the choice of the regularization parameter. Finally, our approach reveals to be insensitive to the ROI size and location.

This is a joint work with S. Bonettini from University of Modena and Reggio Emilia, F. Porta and G. Zanghirati from University of Ferrara.

A Variational Reconstruction Method for Dynamical X-ray Tomography based on Physical Motion Models

Andreas Hauptmann Department of Mathematics and Statistics, University of Helsinki, Finland

Abstract

We study a variational approach that incorporates physical motion models and regularization of motion vectors to achieve improved reconstruction results for dynamical X-ray tomography in two dimensions. We focus on realistic measurement protocols for practical applications, i.e. we do not assume to have a full Radon transform in each time step, but only projections in few angular directions, this restriction enforces a space-time reconstruction.

In particular we utilize the methodology of optical flow, which is one of the most common methods to estimate motion between two images taken at different time instances. The resulting minimization problem is then solved in an alternating way. Results are presented for simulated and real measurement data.

This work is a joint project with: M. Burger, H. Dirks, L. Frerking, T. Helin, and S. Siltanen.

Generalized SART-methods for robust and efficient tomography

Simon Maretzke Institute for Numerical and Applied Mathematics,
University of Goettingen, Lotzestr. 16-18, 37083 Goettingen, Germany

Abstract

In the simultaneous algebraic reconstruction technique (SART), Radon inversion is performed by cyclically updating the recovered object to fit single tomographic projections in each iteration. While the method is computationally cheap and known to exhibit fast semi-convergence for high-quality data sets, it lacks robustness and flexibility to be applicable in more challenging settings. In this work, we derive generalized SART-iterations which enable strong regularization as well as non-quadratic and even non-convex data fidelity terms at unchanged computational complexity. The novel approach constitutes a compromise between the flexibility of variational methods and the efficiency of SART, which we illustrate by tomographic reconstruction examples for cone-beam, heavily corrupted and region-of-interest data sets.

Synchrotron based X-ray tomographic microscopy: A compromise between theory and practise

Federica Marone Swiss Light Source,
Paul Scherrer Institute, Villigen, Switzerland

Abstract

In 1917, Johann Radon proposed the first mathematical formulation of tomographic image reconstruction, namely an exact solution for the reconstruction of a function from its line integrals. This formula is however only accurate for the ideal case, where the line integrals are exact, the projections are continuous and their number is infinite over π.

In synchrotron based X-ray tomographic microscopy, these conditions are actually never satisfied. Multiple sources of error can contaminate the measured line integrals and the measured projections are discrete as a consequence of the pixelated nature of the used detectors. Furthermore, the number of acquired projections is limited. To nonetheless obtain high quality tomographic volumes, it is therefore imperative to condition the measured data using sophisticated algorithms. This is even more true for the steadily increasing complexity of cutting edge tomographic microscopy experiments at synchrotron beamlines pushing spatial, temporal and density resolution, to be achieved with as few measurements as possible. Alternative approaches to this first formulation of tomographic image reconstruction are also often needed, mostly based on iterative processes.

In this presentation we will discuss some typical sources of error affecting the accuracy of the measurements and of the results during daily operation at tomographic microscopy beamlines, including edge-enhancement, local tomography, ring and zinger artifacts, limited number of projections, noise and movement artifacts. We will also present practical and more theoretical solution strategies for mitigation and reduction of the mentioned artifacts.

Three-dimensional reconstruction of a human trabecular bone using sparse x-ray tomography

Zenith Purisha Department of Mathematics and Statistics,
University of Helsinki, Finland

Abstract

The use of a low number of x-ray tomographic data to study the inner structure of a trabecular bone is very crucial in biomedical re- search. Using sparse projection data is desired as it reduces the often long scan times and high ionizing radiation doses in CT. Neverthe- less, when the collection of projection images are only available from the sparse data, the problem becomes ill-posed and the conventional methods such as FDK method are no longer valid. In this work, a shearlet-based strategy is implemented to recover the structure of a human trabecular bone. The shearlet transform is thresholded before computing the inverse shearlet transform to recover the object. Nu- merical examples are demonstrated to reconstruct the bone in three dimension and to study the morphology of the bone. The reconstruc- tions using shearlet-based method outperforms the baseline algorithms such as FDK and SIRT. This project is joint work with Sakari Karhula, Juuso Ketola, Miika T. Nieminen, Simo Saarakkala, Samuli Siltanen.

MS 16: Beyond filtered backprojection: Radon inversion with a priori knowledge

Organizers: Martin Benning, Matthias J. Ehrhardt, and Carola Schönlieb

Direct reconstruction preconditioners for iterative variational Radon inversion

Kristian Bredies University of Graz,
Heinrichstraße 36, 8010 Graz, Austria

Abstract

Variational approaches for Radon inversion allow a flexible choice of regularizers that stabilize the ill-posed problem on the one hand and incorporate a-priori knowledge on the solution on the other hand. Using measure-based smoothness measures such as the total variation (TV) or total generalized variation (TGV) enable, for instance, the recovery of piecewise constant or piecewise smooth functions, even for data that is heavily undersampled in the angular direction. A drawback is, however, that solutions have to be computed via iterative numerical optimization algorithms that usually ask for the evaluation of the Radon transform and the back-projection in each iteration step. This is in contrast to filtered back-projection which performs a reconstruction in basically one iteration step. For this reason, the number of iterations needed by an iterative Radon inversion method is of crucial importance for the practical performance of these methods.

In this talk, we discuss optimization algorithms that incorporate direct reconstruction strategies in order to speed up iterative variational Radon inversion in terms of performing fewer iteration steps. We show that recently-introduced preconditioned Douglas–Rachford iterations are suitable for this purpose and present numerical examples for TV- and TGV-regularized Radon inversion for which preconditioning with direct inversion operators is beneficial in terms of reconstruction speed.

Directional Regularization in CT Reconstruction

Yiqiu Dong DTU Compute, Technical University of Denmark

Abstract

In this talk, I will introduce a new directional regularization based on the total generalized variation (TGV), which is very useful for reconstructing objects whose textures follow certain direction. I will show that it has the many similar essential properties as TGV. With a robust direction estimator, we demonstrate the improvement of using directional TGV compared to standard TGV in the application of computed tomography reconstruction.

Faster PET Reconstruction with a Stochastic Primal-Dual Hybrid Gradient Method

Matthias J. Ehrhardt Department for Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK

Abstract

In this talk we revisit the problem of Positron Emission Tomography (PET) reconstruction with non-smooth and convex priors. As the data fidelity term in PET is the Poisson likelihood there are not many algorithms that can solve this problem. A very popular choice for solving it is the Primal-Dual Hybrid Gradient method proposed by Chambolle and Pock. While this algorithm works well for small to medium size problems in PET, in the case when the data size becomes large computational issues arise. This is due to the fact that the system matrix for clinical PET scanners is very large and cannot be stored in the memory of most computers. Instead an expensive algorithm to compute matrix-vector products has to be employed. In this talk we circumvent this issue by extending the Primal-Dual Hybrid Gradient method to the subset setting (like in ART, Kaczmarz or OSEM). By choosing subsets randomly we can prove that the algorithm is convergent for all sensible random subset selections. Examples based on synthetic and real data show that it is much faster (in terms of actual time) than the standard Primal-Dual Hybrid Gradient method.

This is joint work with Antonin Chambolle, Pawel Markiewicz, Peter Richtárik and Carola-Bibiane Schönlieb.

Solving the Polychromatic inverse problem for X-ray CT

Joshua Greenhalgh Institute of Sound and Vibration Research (ISVR), University of Southampton

Abstract

Prior knowledge can help to solve under-determined inverse problems. We are here interested in one such problem, a non-linear x-ray tomographic inverse problem, where an x-ray source emits photons with different energies, but where the available tomographic measurements are the integral over all energies. The task is the estimation of the energy dependent x-ray absorption profile. We here report on an initial investigation into the types of prior information available and the extent to which these help in solving the inverse problem

It has been known for some time that the assumption of a monochromatic source leads to beam hardening artifacts in x-ray tomographic reconstruction. Our prior knowledge of the true polychromatic nature of emitted photons can be used to form a more realistic forward model. The model constructed using this information is a non-linear relationship between unknown parameters and measurements as opposed to the more familiar linear relationship derived with a monochromatic assumption.

Images are usually piecewise constant - they are formed of many areas of constant value with areas of varying value at edges which are fewer in number. This knowledge about images can be encoded mathematically by noting that the images gradient should be sparse - the $\ell 0$ norm of the gradient should be small. The widely used total variation regularisation term enforces this prior knowledge in our solutions. This particular use of prior information is an example of a more general class - the use of knowledge about not only the image but about the image in a transformed domain.

Solutions to the polychromatic forward model can be expressed as the minimum of non-convex objective function. Due to the non-convex nature of the inverse problem, optimisation is extremely challenging. To explore the nature of this optimisation problem, investigations have been conducted using a range of first order gradient based methods, initially without additional prior information. Minimisation was explored using a toy problem consisting of a single pixel and two energy levels - hence a two variable objective function. Even when this problem was fully determined the minimisation was often found to be difficult due to slow convergence and the apparent existence of multiple minima.

In the more realistic, under-determined, situation of reconstruction from measurements obtained using a simulated polychromatic phantom the optimisation was even more challenging and results exhibited some interesting properties. Convergence to a solution was dependent on the particular energy band and the emitted source spectrum. Energies at which there is high probability of photon emission converged much more quickly to spatially correct solution.

The above results highlight the importance of careful initialisation. For example, the use of a FBP solution for initialisation has proven useful in the past and can allow for the starting point of

the optimisation to be close to a minimum. It is however not obvious how to take a two dimensional reconstruction and use this to create an energy dependent initialisation.

Whilst initial experiments have shown that a non-linear, polychromatic x-ray absorption model is able to reduce beam hardening artifacts in x-ray tomography, the optimisation problem remains challenging. Slow convergence and local minima pose significant challenges to gradient based optimisation and whilst spatial structure can often be estimated well, the nature of the non-convex function means that the accurate estimation of energy dependent absorption profiles remains much more challenging, even if strong prior models are used to constrain the solution.

Shape based prior information in image reconstruction

Ozan Öktem Department of Mathematics,
KTH - Royal Institute of Technology, 100 44 Stockholm, Sweden

Abstract

We will introduce a variational framework for image reconstruction in imaging where shape related a priori information is used. The key element is to use deformable templates machinery from shape theory. Various variational formulations are possible within this framework and we apply som of these to tomographic reconstruction.

Modern Radon inversion from the perspective of industrial X-ray CT

Alex Sawatzky GE Sensing & Inspection Technologies GmbH,
Niels-Bohr-Str. 7, 31515 Wunstorf, Germany

Abstract

Despite an intensive development and acceleration of modern Radon inversion approaches (e.g., non-linear variational methods, primal-dual algorithms, or hardware based acceleration) these continue to be of limited usage in many application areas. In this talk some of these limitations and future requirements will be discussed from the perspective of industrial X-ray CT where the filtered backprojection is still the reconstruction method of choice.

MS 17: Inverse problems for Radiative Transfer Equation and Broken Ray Approximation

Organizers: Linh Nguyen and Markus Haltmeier

A scatter correction algorithm for computerized tomography in semi-transparent media

Herbert Egger TU Darmstadt, Germany

Abstract

The propagation of radiation through semitransparent media is described by the radiative transfer equation. By semitransparent we here mean that the amount of scattered radiation is relatively small in comparison to the transmitted and absorbed fraction. The Radon transform may therefore serve as a first approximation for the radiation transport and computerized tomography (CT) can be utilized to approximately identify absorbing inhomogeneities. In this talk, we discuss the enhancement of such CT images by taking into account the perturbations due to scattered radiation. The resulting algorithms are related to inverse Born series approximations and they allow for an incremental correction of the reconstructed images. An analytical justification of the proposed approach is given and some numerical tests are presented for demonstration.

Pestov identities for generalized X-ray transforms

Joonas Ilmavirta University of Jyväskylä, Finland

Abstract

Pestov identities have proven useful in the study of X-ray tomography on Riemannian manifolds. After discussing previous triumphs of the method, we will extend it to new situations: broken ray tomography and X-ray tomography on pseudo-Riemannian manifolds. We will also discuss the limits of this method. This is partly joint work with Mikko Salo and Gabriel Paternain.

Broken-ray transform and its generalizations

Vadim A. Markel Aix Marseille Univ, CNRS, Centrale Marseille,
Institut Fresnel, F-13013 Marseille, France

Abstract

Recently, there have been a lot of interest in generalizations of the Radon transform that take into account single scattering. Some of these generalizations, which employ energy-resolved measurements, result is transforms of the medium on circles or circular arcs. Some other generalizations, which involve energy-insensitive but angularly-collimated measurements result in the broken-ray (sometimes referred to as V-line) transforms. If the scattering coefficient of the medium is variable, simultaneous reconstruction of scattering and attenuation is possible by combining several V-line measurements with a common vertex. This gives rise to the star transform of the medium. A "star" is a set of rays originating from a single vertex. Directions of the rays are fixed but the vertex can be scanned over the area of a slice. In addition, the integrals along each ray comprising a star can contribute to the data with positive or negative coefficients. I will discuss some of the properties and numerical inversion of the star transform. An interesting result is that inversion of the star transform can be stable only for odd number of rays, which is a necessary condition of stability. Explicit examples of configurations allowing stable inversion can be given. A striking feature of the described imaging modality is that it allows reconstruction of sub-domains of a given slice with incomplete data, that is, when the data are insufficient to reconstruct the functions of interest in the whole slice. Another useful feature of the star transform is that it allows simultaneous and independent reconstruction of the scattering and absorption coefficients of the medium.

Attenuated tensor tomography applied to the source reconstruction problem in transport

François Monard University of California Santa Cruz, USA

Abstract

The problem of Optical Molecular Imaging consists of reconstructing a density of light-emitting molecules from emission measurements outside the region of interest. The propagation model is the radiative transfer equation, describing the transport of photons interacting with a medium via attenuation and scattering effects.

In the absence of scattering, the measured data is the attenuated X-ray transform of the unknown source, a problem whose inversion is now well-understood. In the case of small scattering, one may still set up a convergent iteration scheme based on the inversion of the attenuated X-ray transform, as provided in [Bal-Tamasan, SIAM Math. Anal., 2007]. In this talk, I will explain how the introduction of scattering effects in this problem requires understanding/inverting-modulo-kernel the attenuated X-ray transform over functions which depend on both position and direction. Discussing recent findings on the topic, I will then explain how this can be used to still reconstruct a source accurately, in a regime where the scattering kernel need not be small, but has finite harmonic dependence in terms of the deviation angle.

The source reconstruction problem is joint work with Guillaume Bal and John Schotland.

V-line and Cone Radon transforms: overview and application to cultural heritage object imaging

Mai K. Nguyen Laboratoire Equipes de Traitement de l'Information et Systèmes (ETIS)- ENSEA/ Université de Cergy-Pontoise/CNRS UMR 8051, 2 rue A. Chauvin, F-95302 Cergy-Pontoise, France

Abstract

We present an overview of the so-called V-line and Cone Radon transforms as they arise in Compton scatter imaging and give a short account of the latest developments on this concept in recent years which are relevant for imaging processes in many fields of applications. After recalling their applications in Compton scatter imaging, we show how the two transforms may be supporting a successful cultural heritage object imaging research.

Quantitative Photoacoustic Tomography in the Transport Regime

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Abstract

Quantitative image reconstruction in photoacoustic tomography requires the solution of a coupled-physics inverse problem involving light transport and acoustic wave propagation. The aim is to reconstruct the optical parameters, absorption and scattering, from pressure measurements outside of the sample.

Light propagation in highly scattering media is often described by a diffusion equation. However, especially in the presence of voids or low scattering areas, the radiative transfer equation is a more accurate model. Numerically solving the radiative transfer equation is computationally quite costly due to the higher dimensional nature of the problem. As an intermediate model one can use the radiative transfer equation but linearize the inverse problem in the vicinity of the unknown parameters, absorption and scattering rate.

In this talk we will discuss stability and well-posedness of such a linearization and describe strategies for solving the inverse problem numerically. We compare numerical results for the fully nonlinear and the linearized inverse problem as well as the diffusion approximation.

This talk is based on joint work with M. Haltmeier, S. Rabanser (University Innsbruck) and L. Nguyen (University of Idaho).

Posters

Inverse Problems in Geomagnetism

Christian Gerhards University of Vienna, Computational Science Center

Abstract

In Geomagnetism, typical problems are the reconstruction of magnetizations, susceptibilities, or dipole directions (in the case that the magnetizations are known to be induced by an ambient dipole field) and the separation of the total geomagnetic field into its different contributions (e.g., core and crustal contributions). In this work, we present some examples where the Hardy-Hodge decomposition of spherical vector fields can be used to characterize the reconstructible quantities. In particular, we describe the modeling and separation of the geomagnetic field into its core and crustal contributions in more detail.

Nonlinear Flows for Displacement Correction with Applications in Imaging

Guozhi Dong Computational Science Center, University of Vienna, Austria

Abstract

We derive nonlinear evolution equations associated with a class of non-convex energy functionals which can be used for correcting displacement errors in imaging data. We study properties on the behavior of the solutions of these filtering flows, and provide examples for correcting angular perturbations in tomographical data.

Wavefront Reconstruction in AO using a Singular Value Type Expansion

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Abstract

Atmospheric turbulence and diffraction of light induce blurred images of celestial objects when they are observed by earth-bound telescopes. The new generation of telescopes uses the technology of Adaptive Optics (AO) to correct for the aberrations. An AO system compensates for the rapidly changing optical perturbations arising during the imaging process and physically corrects for atmospheric blurring via deformable mirrors in real-time. Control of a deformable mirror requires the knowledge of the incoming phase. Unfortunately, incoming wavefronts of guide stars cannot be measured directly. Depending on the incorporated wavefront sensor, the connection between the intensity of the incoming light and incoming wavefronts is non-linear in general. Reconstruction of the unknown wavefront from given sensor measurements describes an inverse problem. Furthermore, in the era of Extremely Large Telescopes (ELTs) with primary mirror diameters up to 40 m the computational load of existing control algorithms is immense which makes the development of new and fast methods even more important.

In this poster, we present a new approach of wavefront reconstruction from non-modulated pyramid wavefront sensor data. As a basis for the reconstruction serves a linearization of the forward model represented as the finite Hilbert transform of the incoming phase. Due to the non-compactness of the finite Hilbert transform operator the classical theory for singular systems is not applicable. Nevertheless, we are able to express the Moore-Penrose inverse as a singular value type expansion with Chebychev polynomials.

The effectiveness of the Singular Value Type Reconstruction (SVTR) algorithm is demonstrated in the context of the European Extremely Large Telescope which is currently under construction.

Imaging in Electron Paramagnetic Resonance

Maud KEREBEL Paris Descartes University, France

Abstract

Spatial electron paramagnetic resonance imaging (EPRI) is a recent method to localize and characterize free radicals in vivo or in vitro, leading to applications in material and biomedical sciences. The sinogram provided by this technology is modeled as the noisy convolution of the spectrum of the paramagnetic species with the Radon transform of its concentration. To improve the quality of the reconstruction obtained by EPRI, a variational method is proposed to inverse the image formation model. It is based on a least-square data-fidelity term and the total variation and Besov seminorm for the regularization term. To fully comprehend the Besov seminorm, an implementation using the curvelet transform and the L^1 norm enforcing the sparsity is proposed. It allows our model to reconstruct both image where acquisition information are missing and image with details in textured areas, thus opening possibilities to reduce acquisition times. To implement the minimization problem using the algorithm developed by Chambolle and Pock, a thorough analysis of the direct model is undertaken and the latter is inverted while avoiding the use of filtered backprojection (FBP) and of non-uniform Fourier transform. Numerical experiments are carried out on simulated data, where the proposed model outperforms both visually and quantitatively the classical model using deconvolution and FBP. Improved reconstructions on real data, acquired on an irradiated distal phalanx, were successfully obtained.

This work was jointly undertaken by Sylvain Durand, Yves-Michel Frapart and Maud Kerebel from Paris Descartes University, France, and the proposed talk will display the results submitted for publication in the preprint https://hal.archives-ouvertes.fr/hal-01419832.

Modeling OCT as an Inverse Scattering Problem

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Abstract

In this work we model Optical Coherence Tomography (OCT) as an inverse scattering problem. Given the OCT data, we want to recover the electric susceptibility of a linear dielectric medium. For a dispersive isotropic medium, we present a reconstruction method based on a discretization of the susceptibility with respect to the detection points. This assumption can be neglected if we have additional measurements from another imaging technique. Here, we consider photoacoustic tomography. The iterative scheme can also be applied to the anisotropic case, if we consider additional incident illuminations for different orientations of the sample. This is a joint work with P. Elbau and O. Scherzer.

Quantitative Photoacoustic Imaging in the Acoustic Regime using SPIM

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Abstract

We study the photoacoustic tomography model and we aim to recover simultaneously the absorption density and *acoustic* material parameters of the object such as the sound speed and density. A promising way for simultaneous reconstruction of these parameters is to illuminate just one slice of the object under consideration with a laser pulse so that absorption can only take place in this illumination plane. Using this method of single plane illumination microscopy (SPIM) and asymptotic expansion techniques for the Born approximation, we provide explicit reconstruction of these unknown parameters: absorption density, sound speed and material density.

This is a joint work with Alexander Beigl, Peter Elbau, and Otmar Scherzer.

Recovery of the Lamé parameters in the inverse elasticity problem

Ekaterina Sherina Technical University of Denmark, Kongens Lyngby, Denmark

Abstract

We consider the problem of parameter identification for quantitative elasticity. The inverse problem of elastography for linear isotropic materials is interested in identifying the spatial distributions and values of biomechanical parameters such as the shear modulus and the first Lamé parameter from interior displacement data, which are obtained by ultrasound or magnetic resonance imaging. We assume that the displacement originates from a small quasi-static deformation. For the solution of this ill-posed problem we apply iterative regularization methods and examine their performance on simulated and experimental measurements of the displacement.

This is a joint work with Prof. Otmar Scherzer and Julian Schmid at the Computational Science Center, University of Vienna.

Photoacoustic tomography taking into account acoustic attenuation

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Abstract

In this poster, we will first briefly introduce photoacoustic tomography with attenuation, and then distinguish the attenuation models into strong attenuation and weak attenuation. Based on these two classes, we will present eigenvalue decay rates of the attenuated PAT operator, and reconstruction formulas for the initial pressure.

This is a joint work with Peter Elbau and Otmar Scherzer.

Point spread function reconstruction for Mult-Conjugate Adaptive Optics

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Abstract

In the upcoming generation of Extremely Large Telescopes (ELT), the impact of the turbulent atmosphere is corrected by Adaptive Optics (AO) systems. e.g., by a Multi-Conjugate AO (MCAO) system. Even with AO correction, the quality of astronomical images is still degraded due to the time delay inherent to the system. This results in a blur of the recorded scientific images which can be characterized by the point spread function (PSF). We analyze and present a combined approach for PSF reconstruction based on atmospheric tomography, adapting an algorithm for Single Conjugate AO systems and using the atmospheric layers reconstructed from WFS data. Simulation results obtained in OCTOPUS, the ESO end-to-end simulation tool. suggest a good qualitative performance along with reasonable computational effort.

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